Fig. 1

Fig. 4 *Fig. 3*

Fig. 2 *Engd L.S. Pundersen*

1. Telescopic view of the full Moon. 3. Telescopic view of Saturn & his rings.
2. do do of a part of the Moon near quadrature. 4. do of Jupiter & his Moons.

AN

INTRODUCTION

TO

ASTRONOMY:

DESIGNED AS A

TEXT-BOOK

FOR THE USE OF

STUDENTS IN COLLEGE.

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PREFACE TO THE EDITION OF 1883.

The late discoveries made in Astronomy, principally by the aid of the spectroscope, require that something be added to the descriptive parts of this work. In the present edition, therefore, information of this nature, accompanied with illustrations, is given in an Appendix, with references to and from the corresponding articles in the text.

The mean equatorial Horizontal Parallax of the Sun, adopted from Professor Newcomb's "Investigation of the Distance of the Sun and the Elements which depend on it," is $8''.848$. This number is founded upon a discussion and combination (with their relative weights) of the results given by all the different methods of obtaining the parallax, and therefore is as near an approximation to the truth as can be made at present. The distances and magnitudes throughout the work are reduced to conform to this value.

This edition contains the latest emendations of Professor Snell; and also various numerical corrections, in accordance with the best authorities, for which the Publishers are indebted to Professor Selden J. Coffin, Lafayette College.

Professor Coffin has also added to Art. 264, Appendix M, and has enlarged and thoroughly revised Tables II, IV, and V.

August, 1883.

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ASTRONOMY.

CHAPTER I.

GENERAL FORM AND DIMENSIONS OF THE EARTH.—THE DIURNAL MOTION.—ARTIFICIAL GLOBES.

1. *General definitions.*—Astronomy is the science which treats of the heavenly bodies—that is, of the sun, the planets and their satellites, the comets, and the fixed stars.

The sun, planets, satellites, and comets constitute the *solar system*, which is so called because the sun is the principal body belonging to it, and controls the movements of all the others.

The fixed stars are the bodies situated at vast distances out side of the solar system, and which, on account of that distance, exhibit little or no change of position with respect to each other.

2. *The Copernican system.*—This name is given, in honor of Copernicus, to the science of astronomy as now established by demonstration, in distinction from the erroneous systems of the ancients. It explains the diurnal and annual motions of the heavens, by supposing the earth to rotate each day on its axis, and to revolve once a year around the sun.

3. *The globular form of the earth.*—That the earth is nearly if not exactly a sphere, is indicated in several ways.

1. It is one of the planets. And, as we see the other planets to be nearly spherical, we reason from analogy that the earth is spherical also.

2. In a lunar eclipse, whichever side is turned toward the moon, the outline of its shadow, projected on that body, is always circular.

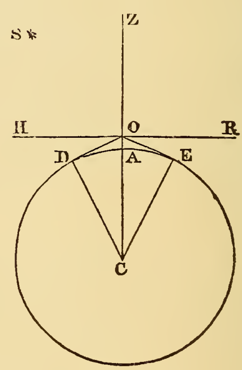
3. Its convexity, by which it wholly or partially conceals distant objects, as a lighthouse or a ship at sea, appears to be equally great on all parts of the ocean.

4. An arc of a given number of miles, measured on any part of the earth, is found always to subtend an angle of nearly equal size at the center; showing that the curvature is everywhere nearly the same.

5. The depression, or dip of the horizon, is equally great at every place, and on every side of the observer, provided his elevation above the ocean level is the same. This will be understood by the next article.

4. *Dip of the horizon.*—If the eye were at A (Fig. 1) on the surface of the earth, the vault of the heavens would be limited by a plane touching the earth at A, and would therefore be just a hemisphere. But if the eye is elevated, as to O, and tangent lines are drawn from that point to the earth on every side, then more than a hemisphere of the sky is visible. Let ZC be the direction of a plumb-line, and let HOR represent a plane perpendicular to it; then there would be a celestial hemisphere in view above this plane, and the remotest visible points on the earth would be depressed below the plane by the angle HOD or ROE. This angle is called the *dip of the horizon.* If AO is a given height, it is found that the angle HOD is sensibly equal on whatever side of the station, or on whatever part of the earth, the measurement is made. It follows from this that the earth is very nearly a sphere.

Fig. 1.



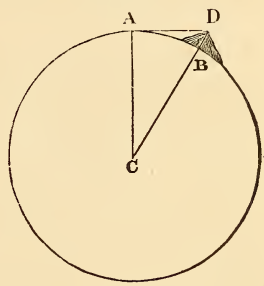
At the height of 100 feet, the depression is about 10′, and varies nearly as the square root of the height.

The word *down* expresses the direction in which a plumb-

line hangs, or a body falls—that is, *toward* the center of the earth. Hence, on different parts of the earth, "down" denotes all possible directions. So "up," or *from* the center, is in every direction; and the direction which is *down* at one place, is *up* at a place on the opposite side of the earth.

5. *Dimensions of the earth.*—The semi-diameter of the earth may be approximately found by measuring the height of the station AO (Fig. 1), and the length of the tangent line OD. If O were the summit of a mountain, then D would be the most distant point from which it could be discerned. In Fig. 2, suppose that the height of the mountain BD, and the distance to the point where it is just seen in the horizon AD, have been measured. Let BD $= h$, and AD $= d$, and the radius, AC or BC $= x$. Then $x^2 + d^2 = (x+h)^2 = x^2 + 2\,h\,x + h^2$. Hence, $2\,h\,x = d^2 - h^2$, and $x = \dfrac{d^2 - h^2}{2h}$.

Fig. 2.



Thus, the semi-diameter of the earth is found in terms of h and d.

The magnitude of the earth may be more accurately found, by measuring the arc of a meridian. Let a line be carefully measured due north on the earth's surface, and the corresponding difference of latitude be observed, as indicated by the change in the elevation of the stars. Then, the surveyed line is the same part of the earth's circumference, which the difference of latitude is of 360°. Thus, if the arc is 1° 30′, its length is found to be about 103.5 miles. Hence,

$$1°\ 30' : 360° :: 103.5 : 24{,}840;$$

which is nearly the number of miles in the circumference of the earth. By a comparison of the most accurate measurements, it is ascertained that

The circumference of the earth	= 24,857 miles.
The diameter (24,857 ÷ 3.14159+)	= 7,912.4 miles.
One degree of the circumference	= 365,000 feet.
One second	= about 100 feet.

6. *Inequalities of surface.*—Although the surface of the earth is uneven, and there are high mountains and deep valleys in many parts of it, yet these are very minute compared with the magnitude of the entire earth; so that the spherical form is not disturbed by their existence. Mountains, four or five miles high on the earth, are relatively no more than are the particles of dust which adhere to a globe one foot in diameter. Thin writing-paper, pasted upon such a globe in the form of the continents, would be sufficiently thick to represent their general elevation above the oceans.

7. *The diurnal rotation.*—The earth revolves continually from west to east, on an imaginary line drawn through its center, called *the earth's axis.* The time occupied in completing a revolution is called a *day*, which is divided into twenty-four hours. A great circle of the earth, perpendicular to the axis, is called the *equator.* In the diurnal rotation, every particle of the earth describes a circle, whose plane is either parallel to the equator or coincident with it. The extremities of the axis are called respectively the *north* and *south poles.*

8. *Secondaries of the equator.*—All great circles passing through the poles, and therefore perpendicular to the equator, are called *meridians.* Such a circle may be supposed to pass through any place whatever on the earth, and is called the meridian of that place. As all great circles of a sphere which are perpendicular to a given great circle, are called its *secondaries*, the meridians are secondaries of the equator.

The *latitude* of a place is its distance north or south from the equator, measured on the meridian of that place, in degrees, minutes, and seconds. *Parallels* of latitude are small circles of the earth, parallel to the equator.

The *longitude* of a place is the distance of its meridian in degrees, minutes, and seconds, east or west from some standard meridian, as that of the observatory of Greenwich. The people of different nations usually reckon longitude from some important observatory of their own country. Thus, the French reckon from Paris, and the Americans from Washington. Any place on the earth is determined by giving its latitude and longitude

9. *The celestial sphere.*—The earth is called the *terrestrial sphere.* The celestial sphere is that apparent vault, called the *sky*, which surrounds the earth on every side, and to which all the heavenly bodies seem to be attached. The center of the earth is regarded as the center of the celestial sphere also. But the distance of nearly all the heavenly bodies is so immense, that it is immaterial from what point of the earth they are viewed. Hence, for most purposes of astronomy, the eye of the observer may be considered as the center of the celestial sphere.

10. *The horizon and its secondaries.*—If the plumb-line (usually called the *vertical*), at any place on the earth, is supposed to be extended till it intersects the celestial sphere, it marks the *zenith* above the place, and the *nadir* below it. And a plane passed through the center of the earth, perpendicular to the vertical, is called the *rational horizon* of that place. This is a great circle of the celestial sphere, and divides it into *upper* and *lower* hemispheres. The *sensible horizon* is parallel to the rational horizon, and passes through the place on the earth's surface. The planes of these two horizons are therefore near 4,000 miles apart; but so great is the distance of the heavenly bodies, that the two planes seem to unite in the same great circle of the heavens.

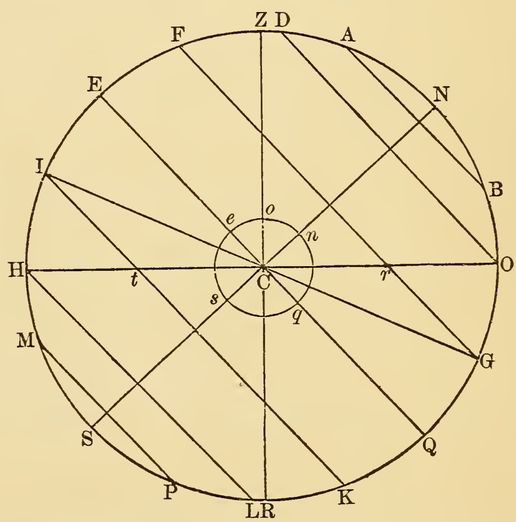
If the observer is at all elevated above the earth's surface, the boundary line between sky and water is a little lower than the horizon, so that somewhat more than half of the celestial sphere is in view (Art. 4). The secondaries of the horizon intersect each other in the vertical line, and are called *vertical circles.* One of them is the meridian of the place. The intersections of the meridian and horizon are the *north* and *south points* of compass. The vertical circle at right angles to the meridian is called the *prime vertical.* This intersects the horizon in the points called *east* and *west.*

The *altitude* of a heavenly body is its elevation above the horizon, measured on the vertical circle passing through the body. The *zenith distance* of a body is the distance between it and the zenith, and is therefore the complement of its altitude.

The *azimuth* of a heavenly body is an arc of the horizon, measured from the meridian to the vertical circle, which passes through the body. The *amplitude* is measured from the vertical circle passing through the body to the prime vertical, and is therefore the complement of the azimuth. The altitude, or zenith distance of a heavenly body, along with its azimuth or amplitude, determines its place in the visible heavens.

11. *The celestial equator and its secondaries.*—If the axis on which the earth revolves is produced to the heavens, it becomes the *axis of the celestial sphere*, and marks the *north and south poles* of that sphere. The north pole is at present in the constellation of Ursa Minor. If the plane of the equator be extended in like manner, it becomes the *celestial equator*. The secondaries to this circle are called meridians, as on the earth. They are also called *hour-circles*, because the arcs of the equator intercepted between them are used as measures of time.

Fig. 3.



Let n (Fig. 3) represent the north pole of the earth, s its south pole, eq the equator (projected in a straight line), o a given

place whose north latitude is *eo*. Then N, S, are the poles of the celestial sphere, EQ is the celestial equator, Z is the zenith of the place *o*, R is its nadir, and HO its rational horizon. *oesqn* is the terrestrial meridian of the same place, and ZESQN is its celestial meridian, or hour-circle.

12. *The ecliptic.*—Besides the equator, there is an important circle of the celestial sphere, called the *ecliptic*. It is that in which the sun appears to make its annual circuit around the heavens. It is inclined to the equator at an angle of nearly $23\frac{1}{2}°$, crossing it in two opposite points, called the *equinoctial points*, or *equinoxes*. The word "equinoxes" is used also to express the *times* at which the sun crosses the equator, because at those times the *nights* are *equal* to the days. The *vernal* equinox is the time when the sun passes the equator from south to north, as it occurs in the *spring*, about March 20th. The *autumnal* equinox occurs on or near September 22d, when the sun returns to the south of the equator.

The *solstitial points*, or *solstices*, are those points of the ecliptic, which are furthest north or south from the equator, situated therefore midway between the equinoxes. They are so named, because there the *sun stops* in his advance northward or southward, and begins to return. The *summer* solstice is the point *where*, and also the time *when* the sun is furthest north, about the 21st of June. He passes the *winter* solstice on or near the 21st of December.

The *equinoctial colure* is that secondary to the equator which passes through the equinoxes. The *solstitial colure* is that which passes through the solstices. They are therefore at right angles to each other, and the latter is a secondary to the ecliptic, as well as to the equator.

13. *Signs of the ecliptic.*—The ecliptic is divided into 12 equal parts of 30° each, called *signs*, which, beginning at the vernal equinox, succeed each other eastward, in the following order:

Northern.		Southern.	
1. Aries . . .	♈	7. Libra . . .	♎
2. Taurus . . .	♉	8. Scorpio .	♏
3. Gemini . . .	♊	9. Sagittarius .	♐
4. Cancer . . .	♋	10. Capricornus .	♑
5. Leo	♌	11. Aquarius . .	♒
6. Virgo . . .	♍	12. Pisces . . .	♓

The vernal equinox being at the first point of Aries, the summer solstice is at the first of Cancer, the autumnal equinox at the first of Libra, and the winter solstice at the first of Capricorn.

14. *Right ascension and declination.*—The right ascension of a heavenly body is the angular distance of its meridian from the vernal equinox, measured eastward on the equator. The declination of a body is its angular distance north or south from the equator, measured on the meridian of the body.

The equator is the plane of reference for right ascension and declination on the celestial sphere, as it is for latitude and longitude on the terrestrial. But terrestrial longitude is reckoned both east and west, while right ascension is reckoned only to the east.

15. *Celestial longitude and latitude.*—On the celestial sphere, longitude and latitude are referred to the ecliptic, not to the equator. Suppose a secondary to the ecliptic to pass through a heavenly body; the distance of the body from the ecliptic, measured on the secondary, is its latitude; and the distance of this secondary from the vernal equinox, measured eastward on the ecliptic, is its longitude.

Right ascension and longitude are reckoned only eastward, from 0° to 360°, the first on the equator, the other on the ecliptic.

16. *Apparent diurnal motion of the heavens.*—As the earth revolves from *west to east* on the axis *ns*, an observer, not being conscious of this motion, sees the heavenly bodies apparently revolving in the opposite direction—that is, from *east to west*, about the axis NS. The sun, moon, and every planet,

comet, and star, is observed to pass over from the eastern part of the sky toward the western, with a regular motion, reappearing again in the east, after the lapse of about one day, in the same, or nearly the same place. The fixed stars describe circles, which are exactly parallel to the equator, and in precisely the same length of time. But the other bodies vary somewhat in their paths, and the periods of describing them, thus indicating that they are affected by other motions besides the diurnal rotation.

17. *Rising, setting, and culmination.*—In Fig. 3, AB, DO, FG, etc., drawn parallel to EQ, represent the diurnal circles of stars, projected in straight lines. Some of these circles intersect the horizon HO. These intersections are the points of *rising* or *setting*. Thus, a star describing the circle GF, rises in the northeast quarter, and sets in the northwest, at points which are both represented by *r*. The star, whose diurnal circle is IK, rises in the southeast, and sets in the southwest, at *t*. A star on the equator rises exactly in the east, and sets in the west, at the point C.

The points, in which these circles cut the meridian, are called the points of *culmination*. Thus, the star on FG makes its upper culmination at F, and its lower one at G. On AB, both the upper and lower culminations are *above* the horizon ; on MP, they are both *below*. If both culminations of a star are above the horizon, it is always in view; if both below, it never comes in sight. The number of stars which do not rise and set, depends on the position of the celestial poles in relation to the horizon—that is, on the latitude of the place.

By the culmination of a body, in the ordinary use of the word, is meant its *upper* culmination.

18. *Relations of the horizon to the diurnal circles.*—Every change of position on the earth changes the horizon. If an observer moves *eastward*, all the heavenly bodies which rise and set, rise *earlier*, and also culminate and set earlier. If he moves *westward*, they rise, culminate, and set *later*. If he moves *toward* the nearer pole of the earth, the corresponding pole of the celestial sphere becomes more elevated, and the

other more depressed; and the contrary, if he moves *from* the nearer pole—that is, toward the equator. In all north latitudes, the north pole is elevated, and the south pole depressed; and the reverse in south latitudes. And the elevation of one pole, and the depression of the other, equals the latitude. For (Fig. 3) NO, the elevation of one pole (=HS, the depression of the other), equals EZ, since each is the complement of ZN But EZ=*eo*, the latitude, because they subtend the same angl at C.

The elevation of the celestial equator equals the complement of latitude. For EH is the complement of EZ, which equals *eo*, the latitude. Hence, the angle by which all the circles of diurnal motion are inclined to the plane of the horizon, equals the complement of latitude, since they are parallel to the equator.

On account of this change of inclination between the horizon and the diurnal circles, the aspect of the diurnal rotation is very different in different parts of the earth.

19. *The right sphere.*—This name is given to those positions, in which the diurnal circles cut the horizon at *right* angles. All points of the equator are so situated. As the latitude is zero, the poles, having no elevation or depression (Art. 18), are both in the horizon; the celestial equator passes through the zenith, thus coinciding with the prime vertical; and all the paths of daily motion, being parallel to the equator, are perpendicular to the horizon. Every heavenly body, unless situated exactly at one of the poles, rises and sets during each revolution, and continues above the horizon just as long as it remains below it. If a star rises in the east, it sets in the west, and culminates in the zenith and nadir.

20. *The parallel sphere.*—This term expresses the appearance of the heavens at those points of the earth where the circles of daily rotation are *parallel* to the horizon. This aspect can be presented only at the poles. For, at those points, the latitude being 90°, one pole must be elevated 90°—that is, to the zenith—and the other depressed 90°, or to the nadir. Hence, the diurnal circles, being perpendicular to the axis, must be

horizontal, and the equator must coincide with the horizon. Every star in view passes around the sky, maintaining the same elevation at every point of its path. No one of the fixed stars ever rises or sets, and every point of a diurnal circle may be regarded as a point of culmination, since it is on a meridian passing through the observer's place.

At the north pole, that half the year in which the sun is north of the equator, is uninterrupted day; during the other half, the sun being south of the equator, it is constant night.

In the right sphere, the whole sky is seen, and every part of it just half the time; in the parallel sphere, only one-half the sky is ever seen, but it is seen the whole time.

21. *The oblique sphere.*—At all latitudes, except 0° and 90°, the circles of daily motion are *oblique* to the horizon, since they incline at an angle equal to the complement of the latitude. Thus, at latitude 42° N., the celestial equator is elevated 48° above the southern horizon, and all the diurnal circles have the same inclination, as shown in Fig. 3. The circle OD, whose distance from the elevated pole equals its elevation, just touches the horizon at the lower culmination, and is the limit of that part of the sky which is always in view. This is called the circle of *perpetual apparition*. The circle HL, at the same distance from the depressed pole, also touches the horizon, and is called the circle of *perpetual occultation*, since it limits that part of the sky which is always concealed.

The horizon HO, bisects the equator EQ. Hence, a body on the equator is as long above the horizon as below it, in every part of the earth. But all bodies between the equator and the elevated pole are longer above the horizon than below, while on the opposite side they are longer below than above.

22. *Artificial globes.*—They are of two kinds, terrestrial and celestial. The terrestrial globe is a miniature representation of the earth, having also the equator and several meridians and parallels of latitude traced upon it. The celestial globe exhibits the principal fixed stars in their relations to each other, and to the equator and ecliptic.

The artificial globe is suspended in a strong brass ring by an

axis passing through the north and south poles, on which it is free to revolve. This ring represents the meridian of any place, and is supported vertically within a horizontal wooden ring which stands upon a tripod. The wooden ring represents the horizon. The brass ring may be slid around in its own plane, so as to elevate or depress either pole to any angle with the horizon. It is graduated from the equator each way to the poles, for measuring latitude and declination; while the horizon ring has near its inner edge two graduated circles, one for azimuth, and the other for amplitude. On this ring also, for convenient reference, are delineated the signs of the ecliptic, and the sun's place in it for every day of the year.

Around the north pole is a small circle, marked with the hours of the day; and at the same pole, a brass index is attached to the meridian, which can be set at any hour of the circle.

The *quadrant of altitude* is a flexible strip of brass, graduated into 90 parts, each equal to a degree of the globe. This can be used for measuring angular distances in any direction on the sphere; and when applied to a vertical circle of the celestial globe, it determines the altitude, or zenith distance of a heavenly body.

To *adjust* either globe for any place on the earth, elevate the corresponding pole to a height equal to the latitude. The axis will then form the proper angle with the horizon. And if the globe is turned (the celestial westward, or the terrestrial eastward), the diurnal motion will be truly represented.

23. *Problems on the terrestrial globe.*

1. To find the latitude and longitude of a place.

   Turn the globe so as to bring the place to the brass meridian; then the degree and minute on the meridian over the place shows its latitude, and the point of the equator, under the meridian, shows its longitude.

   Example. What are the latitude and longitude of New York?

2. To find a place by its given latitude and longitude.

   Find the given longitude on the equator, and bring it to the meridian; then under the meridian, at the given latitude, will be found the required place.

Ex. What place is in latitude 39° N., and longitude 77° W.?

3. To find the bearing and distance of one place from another.

Adjust the globe for one of the places, and bring it to the meridian; screw the quadrant of altitude directly over the place, and bring its edge to the other place. Then the azimuth will be the bearing of the second place from the first, and the number of degrees between them, multiplied by 69½, will give their distance apart in miles.

Ex. Find the bearing of New Orleans from New York, and the distance between them.

4. To find the difference of time at different places.

Bring to the meridian the place which lies west of the other, and set the hour-index at XII. Turn the globe westward, until the other place comes to the meridian, and the index will show the hour at the second place when it is noon at the first. The hour thus found is the difference required.

Ex. When it is noon at New York, what time is it at London?

5. The hour being given at any place, to find what hour it is at any other place.

Find the difference of time between the two places, as in (4); then, if the place, whose time is required, is east of the other, add this difference to the given time; but if west, subtract it.

Ex. What time is it in Boston, when it is 2 P. M. in Paris?

6. To find the antiscii, the periœci, and the antipodes of a given place.

Bring the given place to the meridian; then, under the meridian, in the opposite hemisphere, in the same degree of latitude, are found the antiscii. Set the index to XII., and turn the globe until the other XII. is under the index; then, the periœci will be at the same point of the meridian as the given place was, and the antipodes will be where the antiscii were.

Ex. Find the antiscii, the periœci, and the antipodes of Lake Superior.

To the antiscii, the hour of the day is the same as at the given place, but the season is reversed. To the periœci, the season is the same, but the hour opposite. To the antipodes, both hour and season are opposite

24. *Problems on the celestial globe.*

1. To find the right ascension and declination of a heavenly body.

Bring the place of the body to the meridian; then the point directly over it shows its declination; and the point of the equator under the meridian, its right ascension.

Ex. Find the right ascension and declination of α Lyræ. Also, of the sun on the 3d of May.

2. To represent the appearance of the heavens at any time.

Adjust the globe for the place. (Art. 22.) On the wooden horizon find the day of the month, and against it is given the sun's place in the ecliptic. On the ecliptic find the same sign and degree, and bring the point to the meridian. The globe then presents the positions of the stars at noon. Set the hour-index at XII., and turn the globe till the index points to the required hour. The aspect of the heavens at that hour is then represented.

Ex. Required the aspect of the stars at Lat. 51°, Dec. 5th, at 10 P. M.

3. To find the time of the rising and setting of any heavenly body, at a given place.

Having adjusted for the latitude, bring the sun's place in the ecliptic to the meridian, and set the index at XII. Turn the globe eastward, and then westward, till the given body meets the horizon, and the index will show the times of rising and setting.

The times of the *sun's* rising and setting may be found in the same manner, on the terrestrial globe, since the ecliptic is usually represented on it.

Ex. At what time does the sun rise and set on the 4th of July?

Find the time of the rising and setting of Arcturus on the 10th of November.

4. To find the altitude and azimuth of a star for a given latitude and time.

Adjust the globe for the aspect of the heavens (2) screw the quadrant of altitude to the zenith, and direct it through the place of the star. Then, the arc between the star and the horizon is the altitude; and the arc of the horizon between the quadrant of altitude and the meridian, is the azimuth.

Ex. Find the altitude and azimuth of Sirius, Dec. 25th, at 9 P. M. Lat. 43°.

5 To find the angular distance between two stars.

Lay the quadrant of altitude across the two stars, so that the zero shall fall on one of them; then, the degree at the other will show their distance from each other.

Ex. Find the distance between Arcturus and *a* Lyræ.

6. To find the sun's meridian altitude for a given latitude and day.

Find the sun's place, and bring it to the meridian. The degree over it will show its declination. If the declination and latitude are both north or south, add the declination to the co-latitude; if not, subtract it.

Ex. Find the sun's meridian altitude at noon, Aug. 1st. Lat. 38° 30′ N.

CHAPTER II.

PARALLAX.—ATMOSPHERIC REFRACTION.—TWILIGHT.

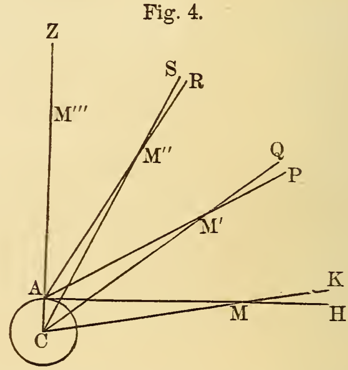
25. *Parallax defined.*—When a person changes his *place,* objects about him in general appear in different *directions* from him. This change of direction is called *parallax.* If, for example, he moves *north*, an object, which was directly *west* or

him, is moved by parallax towards the *southwest;* and an object which was *east*, now appears in the *southeast* quarter. The direction of every thing is more or less altered, except those objects which are in the line of his motion.

26. *Diurnal parallax.*—While a person therefore travels over the earth, or is carried about it by the diurnal rotation, the heavenly bodies must in the same way suffer some parallactic change.

By the true place of a heavenly body, is meant that which it would seem to occupy if viewed from the *center* of the earth. At the *surface*, therefore, it appears generally displaced from its true position; and this displacement is called the *diurnal parallax.* Thus, the true place of the body M (Fig. 4.), is in the direction CK; but at A it appears in the line AH; and the parallax is the angle AMC. So, the true place of M′ is Q, its apparent place is P, and the parallax is AM′C. But the body M‴ appears at Z, whether viewed from A or C, and the parallax in this case is zero. Since the earth's radius, in each instance, subtends the angle of parallax, we have the following definition:

Fig. 4.



The diurnal parallax of a body is the angle at that body subtended by the semi-diameter of the earth.

27. *On what diurnal parallax depends.*—In the triangle ACM′, let AC$=r$, CM′$=d$, and the parallax, AM′C$=p$. Let the zenith distance of the body, ZAM′ $=z$; then, the angle CAM′ is the supplement of z. Hence,

$$\sin p \ : \ \sin z \ :: \ r \ : \ d:$$

$$\therefore \quad \sin p = \frac{r \sin z}{d}$$

Since p is always very small, $\sin p$ varies nearly as p itself

Therefore, regarding r as constant, $p \propto \frac{\sin z}{d}$. That is, *The parallax of a body varies directly as the sine of its zenith distance, and inversely as its distance from the earth's center.*

28. *Horizontal parallax.*—The largest diurnal parallax, which a body can have, occurs when the body is seen in the horizon, as at M. It is then called *horizontal parallax.* From the horizon to the zenith, the parallax diminishes through all values to zero.

In the case of a given body, d is usually constant; and if its parallax, at a certain elevation, has been obtained, its horizontal parallax is found by the variation, $p \propto \sin z$. At the horizon, $z = 90°$, and $\sin z =$ rad. If, when the zenith distance is 53°, the moon's parallax is found by observation to be 45′, then sin 53° : rad :: 45′ : 56′ 21″, which is its horizontal parallax.

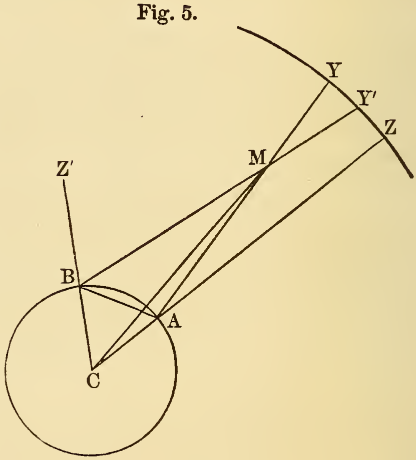
29. *To correct for parallax.*—The effect of parallax is to cause a body to appear *lower* than its true place. Hence, the true altitude of a body is obtained by adding the parallax to its *apparent* altitude.

As parallax is a depression on a vertical circle, then, if a body is on the meridian, the parallax affects its declination just as much as its altitude, since the meridian is also a vertical; bu in other cases, the vertical circle being oblique to the equator the parallax can be resolved into two components, one of which, parallel to the equator, is parallax in right ascension; the other perpendicular to the equator, is parallax in declination.

30. *To find the parallax of the moon.*—Let A and B (Fig. 5) be two stations on the same meridian, taken as far apart as possible. The latitude of each place being known, the arc AB—that is, the angle ACB—is known. When the moon crosses the meridian, let its zenith distance be observed at each station. The observer A sees the moon projected in the sky at Y, and the zenith distance is the angle ZAY, while that at B is Z′BY′. The supplements of these angles, MAC, MBC, are therefore known. In the isosceles triangle ABC, obtain the angles A and B, and the side AB; subtract the angles from

MAC and MBC respectively, then MBA, MAB are known, which, with the side AB, will give AM and BM. Finally, in the triangle AMC, the angle A and sides including it will furnish the angle AMC, which is the parallax sought for the station A, at the zenith distance ZAY. From this the horizontal parallax can be obtained, as in Art. 28.

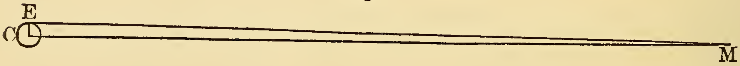
Fig. 5.



The horizontal parallax of the moon is much greater than that of any other heavenly body. Its mean value is about 57′, and is correctly represented by the angle EMC, in Fig. 6.

The above method has also been employed for two or three of the planets, when they come near to the earth. But, with these exceptions, all the heavenly bodies are so far from us, that their horizontal parallax is too small to be obtained in this way with sufficient accuracy. The parallax of the sun is less than 9″; that of nearly all the planets is much smaller than this; and as to bodies outside of the solar system, they afford not the slightest indication of any diurnal parallax.

Fig. 6.

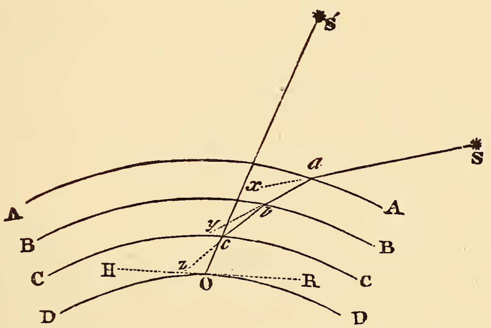


31. *Atmospheric refraction.*—Before the true place of a body can be found by observation, a correction must also be applied for the refraction of its light by the atmosphere. While parallax *depresses* bodies below their true places, more or less according to their distance, refraction *elevates* them, the near and the distant alike.

The earth's atmosphere may be conceived to consist of an

indefinite number of strata, bounded by spherical surfaces, as AA, BB, etc. (Fig. 7), these strata being more dense according as they are nearer the earth. Light from a star S, entering the air at *a*, is bent toward the perpendicular to its surface (which

Fig. 7.



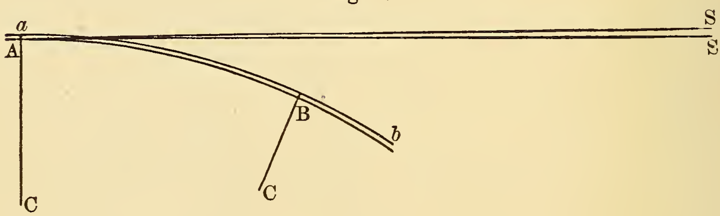
is the earth's radius produced to that point), and describes *ab*, instead of *ax*. For the same reason, it is again bent into *bc*, and then into *c*O; and therefore the star appears in the direction of *c*O produced, at S′, higher than its true place. The path of the ray from *a* to O is in reality not a broken line, as in the figure, but a curve, because the changes of density occur at every point. A body at the zenith is not moved out of place, because its light strikes the surfaces perpendicularly. The refraction at the horizon is about 35′. This is the greatest of all, since the angle of incidence there is the greatest possible. From the zenith to the horizon the refraction constantly increases,—slowly at great elevations, but very rapidly near the horizon, as shown in the following table.

Elevation.	Refraction.	Elevation.	Refraction.
90°	0′ 0″	20°	2′ 37″
80	0 10	10	5 16
60	0 33	5	9 47
45	0 58	2	18 09
40	1 09	1	24 25
30	1 40	0	34 54

The true size of the largest angle of refraction is seen in

Fig. 8. AB is a portion of the surface of the earth, *ab* the surface of the atmosphere, AC, BC portions of the radii of the earth; S is the true place of a star, S′ the place as elevated by horizontal refraction.

Fig. 8.



32. *Measurement of refraction.*—At latitudes greater than 45°, stars which culminate in the zenith make their lower culminations above the horizon. Such a star is observed at both culminations, and its distance from the pole is measured at each. These polar distances are *really* equal, but *apparently* unequal, because below the pole the star is elevated by refraction, while at the zenith it is not displaced. The difference of the apparent polar distances, therefore, gives the amount of refraction at the place of lower culmination.

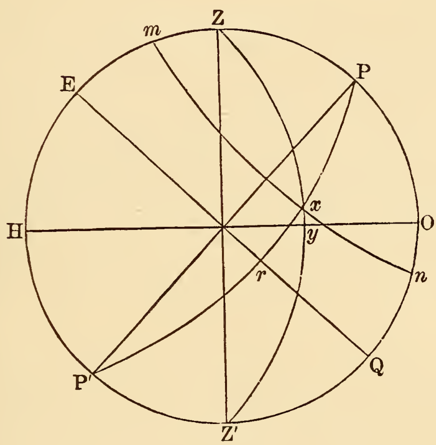
The refraction within several degrees of the zenith is so slight, and its change so uniform, that observations may be made in the same way on stars which culminate several degrees north or south of the zenith; and thus, by applying a small correction, the refraction may be measured at many different altitudes.

33. *General method of measuring refraction.*—A star, whose declination is known, may be used for determining refraction at any altitude, in the following manner.

Let $m\ n$ (Fig. 9) be the path of diurnal rotation of a star, whose declination xr is known. When the star is at x, let its apparent altitude be measured, and let the exact time also be observed. When it culminates at m, observe the time again. The difference of these times, allowing 15° for an hour, will give the angle at the pole Z Px. The co-latitude of the place, ZP, and the co-declination of the star, Px, being known in the

spherical triangle ZPx, the side Zx can be computed. Its complement xy is the true altitude. This, subtracted from the apparent altitude before observed, gives the refraction at that elevation.

Fig. 9.



34. *Tables of refraction.*—It is demonstrated, that except near the horizon, the mean refraction varies as the tangent of the zenith distance. Tables of atmospheric refraction are calculated in accordance with this law, for all zenith distances less than 80°. They are, however, extended beyond that limit down to the horizon, being calculated for the last 10° by a different and more complex law, and the results of calculation being more uncertain. On this account, all astronomical measurements are made, so far as is possible, within 75° of the zenith. In order to obtain the place of a body with the utmost accuracy, tables of refraction are accompanied with means of correcting for the state of the barometer and the thermometer at the time of observation.

35. *Time of rising and setting affected by refraction.*—Since any heavenly body at the horizon is considerably elevated by refraction, it therefore appears to rise earlier and set later

than it would do if there were no atmosphere. The angular breadth of the sun is about 32′, while horizontal refraction is a little more than this—35′. Therefore, the sun appears just above the horizon, when, in truth, it is wholly below. This adds at least four minutes to the day, two in the morning and two at evening.

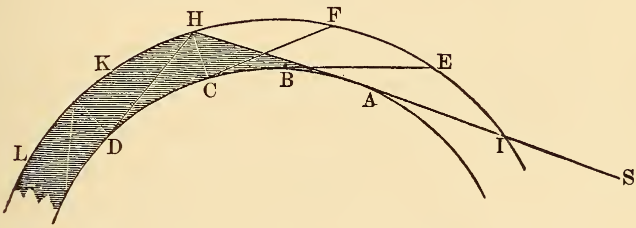
36. *Distortion of the sun's and moon's disk by refraction.*—The change in the amount of refraction is so rapid near the horizon, that when the sun has just risen, or is just about to set, the lower limb is elevated more than the upper, by a very perceptible quantity. Its form, therefore, does not appear circular, but nearly elliptical, the vertical diameter being shortened about 5′ or 6′. The lower half, however, appears more flattened than the upper half, because the difference of refraction between the lower limb and the center is greater than that between the center and the upper limb.

37. *Illumination of the sky.*—During the day, the atmosphere is illuminated by the light of the sun, which penetrates every part of it, and is reflected in all directions. If there were no air, the sky, instead of appearing luminous by day, would exhibit the same blackness as by night, and the stars would be visible alike at all times. We should, in that case, lose a great part of that generally diffused light which illuminates the interior of buildings, and other places screened from the direct rays of the sun. The earth's surface, and all terrestrial objects, on which the sunlight falls directly, would indeed, by radiant reflection, cause a degree of illumination, but it would be far less than we now enjoy. It has been observed, in ascending to great heights, either on mountains or in balloons, where, of course, the air which is most dense and reflects most abundantly is left below, that the sky assumes a very dark hue, and the general illumination is greatly diminished.

38. *Twilight.*—The illumination of the sky begins before the sun rises, and continues after it sets: it is then called twilight. More or less of it is visible, as long as the sun is not more than 18° vertically below the horizon. Those parts of the

atmosphere are most luminous, which lie nearest to the direction of the sun. Thus, in Fig. 10, let A be a place on the earth, where the sun is just setting. The whole sky, IEFH, is illuminated. But, to a place further east, as B, the twilight extends from E to H,—the part of the sky, HK, remote from the sun, being in the shadow of the earth. At C, only FH is illuminated, and HL is dark. At D, the twilight is entirely gone.

Fig. 10.



Though the twilight terminates at H, there is no abrupt transition from light to shade at that point, since the reflection from those high and rare parts of the air is exceedingly feeble; and also, because the thickness of the illuminated segment, through which we look, diminishes gradually to that limit, as is obvious from an inspection of the figure.

39. *Duration of twilight.*—To an observer at the equator, at those times of the year when the sun is on the celestial equator, the twilight continues 1h. 12m. For, in the diurnal motion, 15° are described in an hour, and therefore 18° in $1\frac{3}{15}$h. = 1h. 12m. This is the shortest duration possible. For, if the sun were on a parallel of declination, the degrees of diurnal motion would be shorter than those on a great circle. And, if the observer were on some parallel of latitude, the circles of daily motion would be oblique to his horizon, and the sun must therefore pass over more than 18°, in order to move 18° vertically. An extreme case occurs at the poles, where twilight lasts several months.

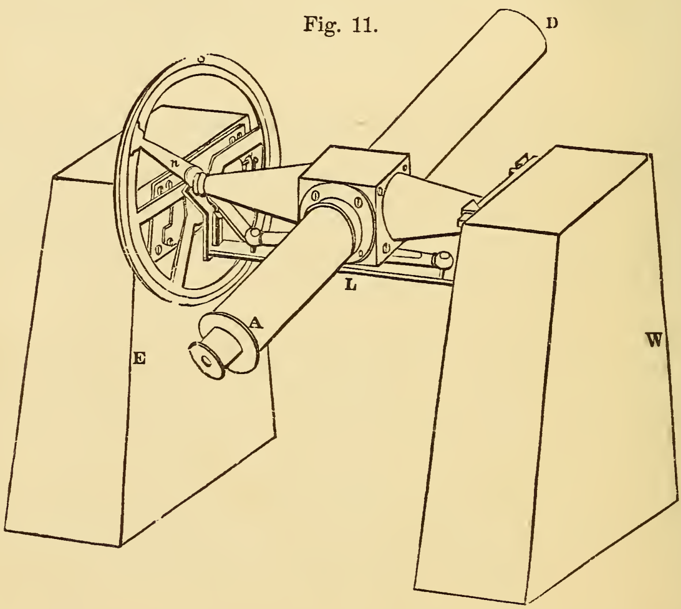
CHAPTER III.

THE OBSERVATORY AND ITS INSTRUMENTS.—SPHERICAL PROBLEMS.

40. *The observatory.*—Accurate knowledge of the motions of the heavenly bodies is mostly obtained by observing their relations to the diurnal rotation. The observatory is furnished with several instruments by which such observations are made.

41. *The transit instrument.*—This is a telescope so mounted as to observe a heavenly body, at the instant when it culminates—that is, makes a *transit* of the meridian. AD

Fig. 11.

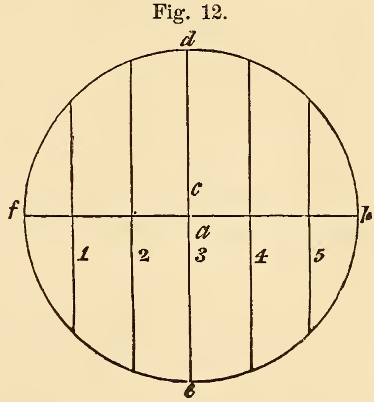


(Fig. 11) represents the telescope supported by a horizontal axis, which consists of two hollow cones, placed base to base, so as to combine lightness and strength. The ends of the axis rest in sockets, attached to two stone piers, E and W. That

the instrument may receive no tremors from the building, the piers stand on a firm foundation in the ground, passing through the floor without contact. The axis being placed east and west horizontally, the telescope, which is perpendicular to it, will, when turned, revolve in the plane of the meridian. A graduated circle, n, is attached to one end of the axis, for marking altitudes or zenith distances. The whole instrument can be raised from the sockets, and the axis inverted, so that the east end shall rest on the pier W, and the west end on the pier E.

42. *Adjustments of the transit instrument.*—The visual axis of the telescope, AD, is called the *line of collimation*, and is marked by the intersection of two exceedingly fine wires in the focus of the eye-glass. One of these wires is horizontal, fh (Fig. 12), the other vertical, de; the latter visibly marks the direction of the meridian, when the instrument has been properly adjusted. The sockets, in which the ends of the axis rest, are so connected with the stone piers, that one of them can be raised or lowered by a screw, and the other can, in a similar manner, be moved north or south. By the spirit-level, L, which hangs on the axis, it can be seen whether the axis is horizontal. If not, raise or lower the end which admits of vertical motion. To find whether the line of collimation is perpendicular to the axis of revolution, observe whether a distant terrestrial object, which is on the vertical wire, remains on it after the ends of the axis have been inverted in their sockets. If not, move the plate which carries the wires laterally, till the vertical wire bisects the distance between the two positions of the object. And finally, to determine whether the axis is east and west, observe if a circumpolar star occupies the same length of

Fig. 12.



time in passing from the upper to the lower culmination, as from the lower to the upper; and if not, move the end of the axis horizontally, till the intervals are equal.

For fuller instructions on adjustment, see Loomis's Practical Astronomy.

43. *The astronomical clock.*—The transit instrument marks the *event* of crossing the meridian; the clock must be used in connection with it, to fix the *time* of the transit. The clock of the observatory is made to keep *sidereal* time,—that is, it marks off 24 hours in the interval between two successive transits of a *star*, instead of the sun. This interval is called a *sidereal day*, and is about 4 minutes less than a solar day. The sidereal day begins when the vernal equinox transits the meridian. At that instant, the clock is at 0h. 0m. 0s.; and any hour of the clock shows how long a time has elapsed since the equinox culminated.

44. *Error and rate of clock.*—The *uniform* movement of the clock is its most important excellence. This may be tested by the transit instrument, and a list of right ascensions of stars. If it does not indicate 0h. 0m. 0s. when the vernal equinox culminates, the difference is called its *error*. If it marks any more or less than 24 hours between two successive transits of a star, this gain or loss is called its *rate*. If both error and rate are known, then the true time is known; and generally it is not best to alter the clock, but only to keep a record of error and rate.

45. *To observe the right ascension of a heavenly body.*—Having elevated the telescope to the altitude of the body at the time of culmination, notice the exact instant when it appears on the vertical wire *de* (Fig. 12). This is its right ascension, which may be given either in time or in arc. Thus, if the clock is at 13h. 46m. 32s. when a star passes the wire, its right ascension is 13h. 46m. 32s.; or, at the rate of 15° for each hour, 206° 38′ 0″.

To secure greater accuracy, several equidistant wires are placed parallel to *de*, an equal number on each side, as in Fig

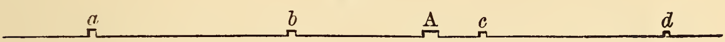
12. The time of passing each wire is noted, and the average of all obtained for the time of crossing the central one.

To observe the right ascension of the sun or a planet, the transit of each limb must be noticed, and the mean of all the times will be the right ascension of the center of the disk.

In order to render the wires visible by night, the field of view is faintly illuminated by a lamp, placed at one end of the hollow axis, the light of which, after entering the telescope, is reflected toward the eye-piece.

46. *Transits recorded by the chronograph.*—To observe the time of a star-transit, the *eye* must discern the instant of its bisection by the wire, and the *ear* must hear the beat of the clock,—the seconds being counted from the last completed minute before the observation began. If the bisection occurs between two beats, as it commonly does, the observer needs much practice to be able to divide the second accurately into tenths, and decide at which of them the transit takes place. Transits are now generally observed and recorded with much greater ease and accuracy by the use of the galvanic circuit.

Fig. 13.

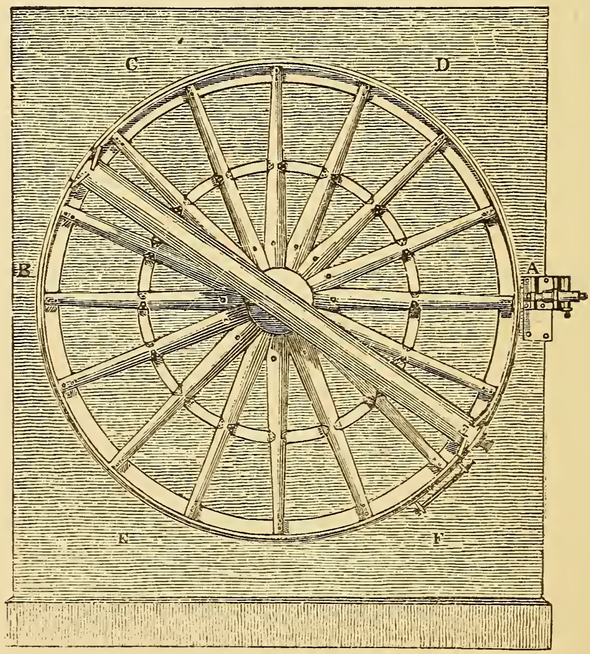


The pendulum of the observatory clock is arranged to close the circuit of a battery and break it again, at the beginning of every beat. The closing of the circuit gives a small lateral motion to the registering pen, under which the paper is advancing on a revolving cylinder, about an inch per second. Thus the seconds are all permanently recorded by notches one inch asunder in a straight line, as *a*, *b*, *c*, *d* (Fig. 13). The mark at the beginning of each minute has some peculiarity by which it may be distinguished from the rest. The observer has under his hand a key, which, by a quick touch, will also close and break the circuit. Whenever a star is on one of the wires of the transit instrument, he touches the key, the pen is moved aside, and indents the line as at A, and the observation is thus recorded; and the place where this motion commenced between the second-marks can afterward be carefully examined. Thus,

without the distraction of attending to the clock, he can record the transits of all the wires; and if he only notices within what *minute* the work begins, he can read the entire record with accuracy to the $\frac{1}{10}$ or even the $\frac{1}{100}$ of a second. Since the general adoption of this method, the number of wires has been increased, sometimes to 30 or 40, so as to obtain the mean of more numerous observations on the same star. The instrument, as above described, is known as the *chronograph.*

47. *The mural circle.*—The circle of the transit instrument is used principally for finding a body whose altitude is known, and is too small for accurate measurement of arcs on the meri-

Fig. 14.



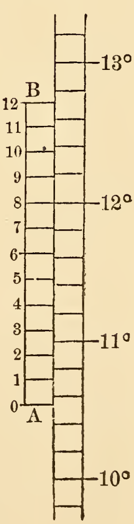
dian. For measuring meridian arcs, the *mural circle* is employed; so called, because it revolves by the side of a vertical *wall.* It consists of a circle usually six or eight feet in diameter, and a telescope attached to its face. It is made so large, in

order that very small angles may be measured by the divisions on its limb. Fig. 14 represents the instrument attached to the meridian wall. Its radii are hollow and of conical form. The axis, which is on one side only, is firmly set in the wall; and the circle and telescope revolve upon it. The graduations are made on the rim, and not on the face of the circle, and are read by means of microscopes attached to the wall.

48. *Subdivisions of the graduated limb.*—The reading of a graduated arc can always be carried much lower than the divisions actually marked on it. This is sometimes accomplished by the vernier, and sometimes by the reading microscope.

49. *The vernier.*—This contrivance, so named from the inventor, is a short graduated arc, which slides along the limb of the circle that is to be subdivided. For example, AB (Fig. 15) is a vernier for dividing the 12′ spaces of the arc on its right into portions of 1′ each. For this purpose, the vernier consists of 12 parts, which together are equal to 11 of the divisions of the limb. Since 12 parts of the vernier are less than 12 divisions of the arc by a whole division, *one* part of the vernier is less than *one* division of the arc by $\frac{1}{12}$ of a division; *two* are less than *two* by $\frac{2}{12}$ of a division, and so on. Now, in the figure, the zero of the vernier has passed 10° 24′; and in order to find how many twelfths of the next space it has passed, it is only necessary to look along the vernier, and observe the number of the division line, which coincides with a line of the arc. In this case we find it to be the 8th. Hence, the 8 parts of the vernier from 0 to 8 are less than the corresponding 8 divisions of the arc by $\frac{8}{12}$; that is, zero is $\frac{8}{12}$ of 12′ beyond 10° 24′. Therefore the reading is 10° 32′.

Fig. 15.



The vernier is sometimes made, so that a given number of parts equals one *more*, instead of one *less*, than the same number on the limb. But the principle of making subdivisions is the same.

50. *The reading microscope.*—This is a compound microscope, having in the focus of its eye-piece a pair of spider-lines intersecting each other, and in the same field of view are the magnified divisions of the arc. The intersection of the spider-lines is moved laterally from one division line of the arc to another by a screw. If the divisions, for example, are equal to 5′ each, then the screw is so made as to move the intersection from one line to another by five revolutions, and therefore each revolution indicates a motion of 1′. A circle is attached to the axis of the screw, having its circumference divided into 60 equal parts. As each revolution of the screw can thus be divided into 60 equal parts, so each minute of the arc can be divided into seconds.

One of these reading microscopes is represented at A (Fig. 14); and the places of others are marked at B, C, D, E, F, 60° from each other. *Six* are used, instead of one, for the purpose of obtaining a more accurate result, by taking a mean of the seconds in the several readings.

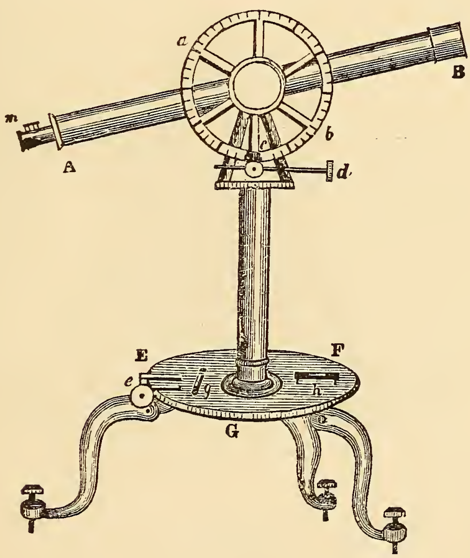
51. *To find the declination of a heavenly body.*—This may be done by measuring its meridian altitude. Let the mural circle be adjusted in altitude, so that, at the instant when the body crosses the vertical wire of the telescope, it is on the horizontal wire also. The graduation of the limb shows its altitude. The latitude of the observatory being known, the elevation of the equator is known ; and the difference between the altitude of the body and the elevation of the equator, is the declination sought. In northern latitudes, if the altitude of the heavenly body exceeds the elevation of the equator, the difference is a northern declination ; if it is less, the declination is south.

Before altitudes can be measured, the horizontal position of the telescope must be determined. This may be done by bisecting the angle between the direction of a fixed star, as seen at culmination, and its apparent direction, when seen at another culmination in a mirror of liquid mercury, called the artificial horizon. By a law of optics, the apparent depression below the horizon equals the elevation above it, so that the whole angle equals twice the altitude.

52. *The transit circle.*—Sometimes the circle of the transit instrument is made of much larger size than is represented in Fig. 11, in order that declinations as well as right ascensions may be observed by it. This combination of the transit instrument and mural circle is called the transit circle, and is considered by some practical astronomers to possess an advantage over the mural circle in the steadiness of its axis.

53. *The altitude and azimuth instrument.*—The essential parts of this instrument are, a telescope and two graduated circles, one vertical, the other horizontal. Fig. 16 presents one of its more simple forms. The telescope AB is movable on a

Fig. 16.

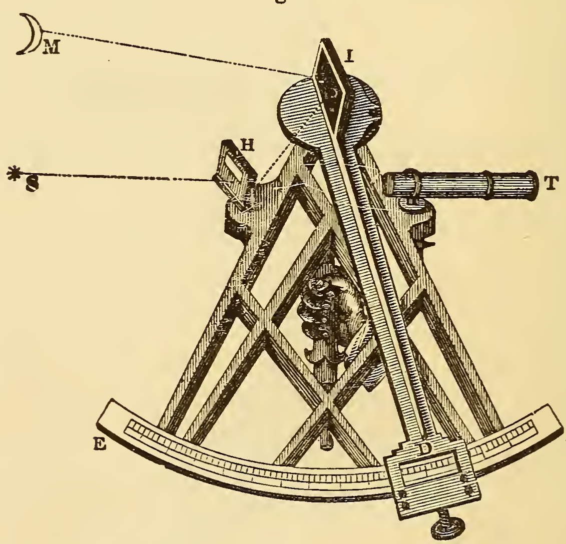


horizontal axis at the center of the vertical circle *abc*, and also on a vertical axis, passing through the center of the horizontal circle EFG. The levels *g* and *h*, placed at right angles to each other, show when the circle EFG is brought to a horizontal position by the tripod screws. The tangent screws, *d* and *e*, give slow motions, one in a vertical, the other in a horizontal plane. If the reading of the vertical circle is taken when the

telescope is horizontal, and again when it is directed to a star, the difference of the readings is equal to the altitude of the star. In a similar manner, if the horizontal circle is read, when the telescope is directed to the north, and read again when it is directed to a star, the difference is its azimuth.

54. *The sextant.*—This is an instrument for measuring the angular distance between two points situated in any plane whatever. It is represented in Fig. 17. I and H are two small mirrors, and T a small telescope. ID is a movable radius or index, carrying the *index mirror* at the center of motion, I,

Fig. 17.



and a vernier at the extremity, D. The *horizon glass*, H, is silvered only on one-half of its surface. When the zero of the vernier coincides with that of the arc at F, the mirrors are precisely parallel. If now we direct the telescope to a star, it may be seen in the transparent part of the horizon glass, and its image in close contact with it, in the silvered part. This is owing to the fact, that a heavenly body is so far distant, that the rays from it to the two mirrors are sensibly parallel to each other.

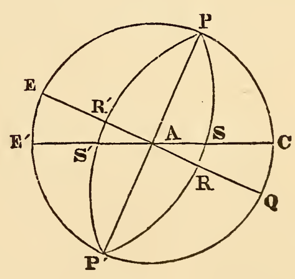
55. *To measure an angle by the sextant.*—Let it be required to measure the angular distance between the star S and the moon M. The telescope being directed to S, and the sextant being held so that the plane of reflection shall pass through the two objects, turn the index from F toward E, until the image of the moon is brought to the star, its nearer limb just touching S. Now, according to an optical principle, the angular distance between the moon and its image is just twice that between the mirrors. Therefore, by reading the vernier at D, we obtain the angular distance between the star and the moon's nearer limb. Again, bring the further limb to the star, and find its distance. Half their sum is the angular distance between the moon's center and the star.

In like manner, the altitude of a body may be found, by bringing its image to coincide with the image of the same body seen in the artificial horizon. One-half the angle read from the vernier is the altitude of the body.

The graduation on the limb of the sextant, for convenience, corresponds, not to the actual length of the arc passed over by the vernier, but to the angular motion of the body, which is twice as rapid. Hence, on the arc of 60°, the graduation reaches 120°; and all angles not greater than this can be measured by the instrument.

The two instruments just described are sometimes convenient at the observatory, but their chief use is elsewhere. The altitude and azimuth instrument is of great value in trigonometrical surveying. The sextant is important for the navigator, since a stationary instrument cannot be employed at sea.

Fig. 18.



56. *Spherical problems.*—

I. To compute the sun's right ascension, declination, or longitude, or the obliquity of the ecliptic to the equator, when any two of the others are given.

Let PEP′ (Fig. 18) represent the solstitial colure, PP′ the axis, EQ the equator, E′C the ecliptic,

and PSP′ a secondary of the equator passing through the sun S. Then SAR is the obliquity of the ecliptic, and RS the declination of the sun. And if its longitude is less than 90°, AS is its longitude, and AR its right ascension. If its longitude is more than 90°, AS and AR are the supplements of longitude and right ascension. In both cases the declination is north. When the sun's place is represented by S′, and its longitude is between 180° and 270°, then the longitude $=180° + AS'$, and the right ascension $=180° + AR'$. But if its longitude is more than 270°, longitude $=360° - AS'$, and right ascension $=360° - AR'$. In each case the declination is south.

The triangle ARS is right-angled at R; and by Napier's rule, any one of the parts may be found, when two others are given.

Ex. 1. When the sun's right ascension is 53° 38′, and its declination, 19° 15′ 57″, required its longitude, and the obliquity of the ecliptic.

1. Rad . cos AS = cos AR . cos RS.

2. Rad . sin AR = tan RS . cot A.

Ans. Long. = 55° 57′ 43″. Obl. = 23° 27′ 50½″.

Ex. 2. On March 31st, the sun's declination was observed to be 4° 13′ 31½″, and the obliquity was 23° 27′ 51″; required the sun's right ascension. *Ans.* 9° 47′ 59″.

Ex. 3. What is the sun's longitude in November, when its declination is 21° 16′ 4″, and its right ascension is 16h. 14m. 58.4s.? *Ans.* 245° 39′ 10″.

The above data show that the sun's longitude is more than 180° and less than 270°, and the declination south. The triangle for computation is AR′S′.

Ex. 4. The sun's longitude being 8^s 7° 40′ 56″, and the obliquity 23° 27′ 42¼″; required right ascension in time.

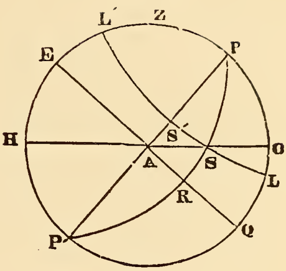
Ans. 16h. 23m. 34s.

II. Given the latitude of a place, and the declination of the sun, to find the time of its rising and setting.

Let PEP′ (Fig. 19) be the meridian of the place, Z its zenith, and HO its horizon. Let LL′ be the diurnal circle of the sun; RS is its declination, S the place of its rising and setting, and LS the arc described between either and midnight. But LS, in degrees, equals QR, the complement of AR. The angle

SAR = EAH, which is measured by EH, the co-latitude, and R is a right angle. Therefore, rad . sin AR = cot A . tan RS.

Fig 19.



Ex. 1. Required the time of sunrise at latitude 52° 13′ N., when the sun's declination is 23° 28′ N.

We find AR = 34° 3′ $21\frac{1}{4}''$; ∴ QR = 55° 56′ $38\frac{3}{4}''$ = (in time) 3h. 43m. $46\frac{1}{2}$s. This is the time of sunrise. The same subtracted from 12h., gives 8h. 16m. $13\frac{1}{2}$s. for the time of sunset.

Ex. 2. Required the time of sunrise at latitude 57° 2′ 54 N., when the sun's declination is 23° 28′ N.

Ans. 3h. 11m. 49s.

Ex. 3. How long is the sun above the horizon in latitude 58° 12′ N., when its declination is 18° 40′ S.?

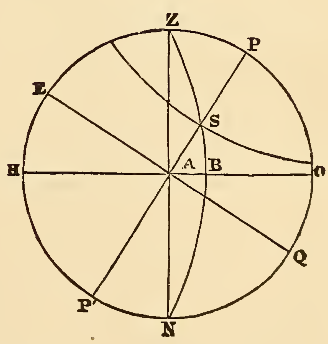
Ans. 7h. 35m. 52s.

In a similar manner, if the declination of any heavenly body be given, the interval of time between its culmination, and its rising or setting, can be computed.

III. Given the latitude of a place, and the declination of a heavenly body, to compute its altitude and azimuth, when on the six o'clock hour-circle.

Let PEP′ (Fig. 20) be the meridian of the place, and P the elevated pole. Then PP′ represents the six o'clock hour-circle, which is at right angles to the meridian, and therefore projected in a straight line. Let the body cross it at S, and let ZSB be the vertical circle passing through it. In the triangle ASB, AS is the declination, SB the altitude, AB the amplitude or complement to the azimuth OB, and B is a right angle.

Fig. 20.



Ex. 1. What were the altitude and azimuth of Arcturus,

when on the six o'clock hour-circle, latitude 51° 28′ 40″ N., its declination being 20° 6′ 50″ N.?

Ans. Altitude 15° 36′ 27″; Azimuth 77° 9′ 4″.

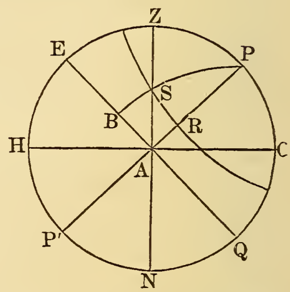
Ex. 2. In latitude 62° 12′ N. the altitude of the sun at six o'clock, A. M., was observed to be 18° 20′ 23″. Required its declination and azimuth.

Ans. Declination 20° 50′ 12″ N.; Azimuth 79° 56′ 4″.

IV. Given the latitude of a place and the sun's declination, to find the time A. M. when it will cease shining on the north side of a building, or the time P. M. when it will begin to shine upon it.

Let PEP′ (Fig. 21) be the meridian of the place, ZAN the prime vertical, and S the place where the sun crosses it, and thus ceases to shine on the north side of a vertical wall. Let PSB be the hour-circle through the sun at S. BS is the sun's declination, BAS (=EZ) is the latitude, and AB, changed into time, will show how long after six o'clock A. M., or before six P. M., the sun transits the prime vertical.

Fig. 21.



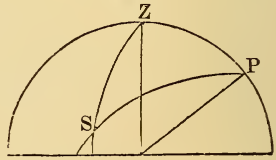
Ex. 1. In latitude 42° 22′ 17″ N., when the sun's declination is 23° 27′ 36″ N., at what times does the sunshine begin and end on the north and south sides of a building? *Ans.* 7h. 53m. 38s. A. M., and 4h. 6m. 22s. P. M.

Ex. 2. How long does the sun shine on the south side of a vertical wall, in latitude 20° 30′ N., when the sun's declination is 20° N? *Ans.* 1h. 45m. 48s.

V. The latitude and the sun's declination being given, to find the time of day by the sun's altitude.

Let Z (Fig. 22) be the zenith of the place, P the pole, and S the place of the sun. Measure ZS, the zenith distance of the sun, and correct it for refraction and parallax. PZ is the co-latitude of the place, and PS the co-declination of the sun. Therefore, the sides of the spherical triangle PZS are all known, and the angle ZPS can

Fig. 22.

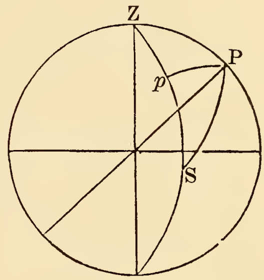


be computed; which, changed to time, shows how long before or after noon the observation was made.

VI. Given the latitude and the sun's declination, to find the time when twilight begins and ends.

The twilight begins or ends when the sun is about 18° below the horizon (Art. 38). Let Z (Fig. 23) be the zenith, P the pole, and S the place of the sun at the beginning or end of twilight. ZS = 108°, ZP = co-lat., PS = co-decl. The three sides of ZPS are given, to find the hour-angle ZPS. This may be done by dropping the perpendicular arc Pp, and using the proportion (Sph. Trig.) $\tan \frac{1}{2}$ ZS : $\tan \frac{1}{2}$ (PS + ZP) : : $\tan \frac{1}{2}$ (PS − ZP) : $\tan \frac{1}{2}$ (Sp − Zp). Having obtained Zp and Sp, compute the angles at P, and add them together.

Fig. 23.



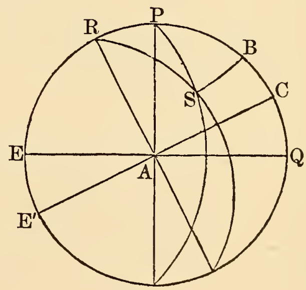
Ex. In lat. 42° 22′, when does twilight begin and end, at midsummer, the sun's declination being 23° 28′?

Ans. 2h. 6m. 20s. A. M.; 9h. 53m. 40s. P. M.

VII. Given the right ascension and declination of a body, to find its longitude and latitude.

Let EQ (Fig. 24) be the equator, and P its north pole, E′C the ecliptic, and R its pole, and S the place of the body. Join PS and RS, and draw the arc SB perpendicular to PC. PS, the complement of declination, is known; likewise RP, which equals EE′, the obliquity. As A is the vernal equinox, SPQ is the complement of right ascension, and therefore known. SRC is the complement of longitude, and RS is the complement of latitude.

Fig. 24.



In the right-angled triangle PSB, PS and P being known, find PB. Then RB (= RP + PB) is known. Then (Sph.

Trig.) sin RB : sin PB :: tan P : tan R. Thus R, the complement of longitude, is found. Then, in the right-angled triangle RSB, RB and the angle R enable us to find RS, the complement of latitude.

Ex. 1. The right ascension of a planet was observed to be 82° 7′, and its declination 24° 26′ N. Calling the obliquity 23° 27′ 20″, what were the longitude and latitude of the planet? *Ans.* Long. 82° 49′ 30″; Lat. 1° 10′ 27″ N.

Ex. 2. What are the longitude and latitude of the star, whose right ascension is 4h. 40m. 49s., and its declination 66° 6′ 37″ N.? *Ans.* Long. 79° 7′ 8″; Lat. 43° 24′ 5″ N.

CHAPTER IV.

THE EARTH'S ANNUAL MOTION ABOUT THE SUN.—THE SEASONS.—FIGURE OF THE EARTH'S ORBIT.

57. *Observations of the sun's place.*—If we employ the instruments of the observatory in measuring from day to day the right ascension and declination of the sun, at the moment of its crossing the meridian, it will be discovered that these quantities are constantly changing; or, in other words, that the sun is constantly shifting its place in relation to the stars.

58. *Its right ascension.*—By the transit instrument and clock, it is found that the sun's right ascension is always increasing by a quantity which is not quite uniform, but which amounts to nearly one degree every day. So that, in about 365 days, it describes the whole 360° of right ascension, and appears again in the same place among the stars. This is the apparent annual motion of the sun, by which it seems to pass round the heavens from west to east once in a year.

59. *Its declination.*—But while thus passing round, it also moves alternately north and south. For, by measuring the declination each day by the mural circle, it is found that after passing the vernal equinox, March 20th, its declination is north, and increases to the summer solstice, June 21st, when it reaches nearly $23\frac{1}{2}°$; from that point it diminishes to zero at the autumnal equinox, September 22d. The declination then becomes south, increasing to the winter solstice, December 21st, when it is $23\frac{1}{2}°$, and thence diminishing to nothing at the vernal equinox, on March 20th of the following year.

60. *The ecliptic.*—The apparent annual path of the sun is found by the foregoing observations to lie in a *plane*, cutting the celestial sphere in a circle called the *ecliptic* (Art. 12), and inclined to the plane of the equator at an angle of about 23° 27′. This plane maintains almost a constant position among the stars, and is used far more than any other circle of the sphere as a plane of reference.

The obliquity of the equator to the ecliptic in 1850 was 23° 27′ 31″, and diminishes at the rate of 46″ in a century.

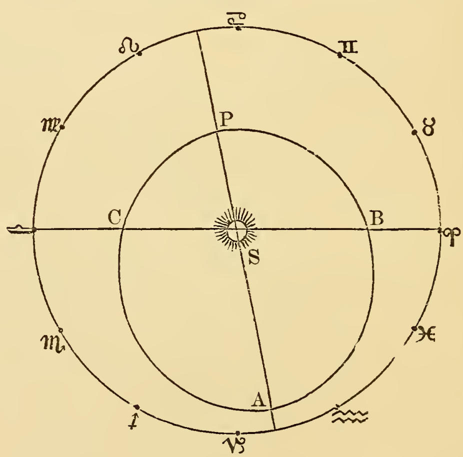
61. *The zodiac.*—This name is given to a zone of the heavens, 16° wide, extending along the circle of the ecliptic, 8° on each side of it. The paths of the principal planets lie within this zone. Its length is divided into 12 signs of 30° each, having the same names and arranged in the same order as those of the ecliptic (Art. 13), though not coincident with them. The signs of the zodiac are distinguished from each other by the stars which occupy them.

62. *The tropics and polar circles.*—Through the two points of the ecliptic most distant from the equator, called the solstices (Art. 12), we imagine circles to be drawn parallel to the equator, called the *tropics.* The northern circle, passing through the first of Cancer on the ecliptic, is called the tropic of Cancer; the southern one, for a like reason, is called the tropic of Capricorn. Two other parallels to the equator, passing through the poles of the ecliptic, and therefore 23° 27′ from the poles of the equator, are called the *polar circles.*

63. *Terrestrial zones.*—On the terrestrial sphere, a similar system of circles divides the earth's surface into the well-known zones of geography, called the *torrid*, *temperate*, and *frigid* zones. The tropics are the limits of vertical sunshine in mid-summer. The polar circles are the limits within which the sun makes a diurnal revolution in midsummer and mid-winter, without rising or setting

64. *The annual motion observed without instruments.*—If the stars were visible in the daytime, we should perceive the sun making progress among them toward the east, by a distance equal to nearly twice its own breadth every day, since the apparent diameter of the sun is a little more than half a degree. But, as they are invisible by day, we detect the same fact, when we notice that at a *given hour* of the night, all the stars are further west than on a previous night. For example, at 9 o'clock P. M.—that is, 9 hours after noon—it is easily observed that there is, from one evening to another, a regular progress of all the stars westward, as long as we choose to watch them. In other words, the sun is at the same rate advancing *eastward* relatively to the stars.

Fig. 25.

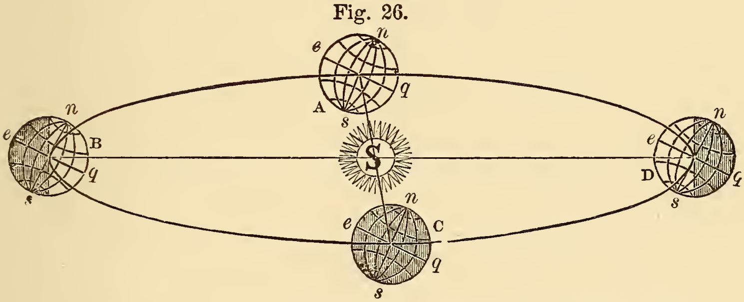


65. *The annual motion is a motion of the earth, not of the sun.*—There is abundant evidence that the motion of the sun

around the earth, above described, is only *apparent*, and results from a *real* motion of the earth about the sun. Thus, suppose the earth to pass around the sun S (Fig. 25) in the orbit ABPC, in the order of the signs; if we were unconscious of this motion, the sun would appear to us to move about the earth in the *same order* of the signs, though, at any given moment, in a *contrary direction.* When the earth is at B (in the sign ♈, as seen from the sun), we should see the sun in the sign ♎; when we reach ♉, the sun is seen in ♏; and so on.

66. *Cause of the change of seasons.*—The changes of the seasons are due to the fact, that the two revolutions of the earth, one on its axis, and the other around the sun, are in different planes; in other words, that the equator and the ecliptic make an angle with each other. In Fig. 26, let ABCD represent the ecliptic (seen obliquely), and suppose the earth to pass around in the order of the letters, occupying the position A on the 20th of March, B on June 21st, C on Sept. 22d, and D on Dec. 21st. In every position of the earth, the equator *eq*, is inclined the same way, and always at the angle of 23½° with the ecliptic. The axis *ns*, being perpendicular to the equator, is everywhere parallel to itself.

Fig. 26.



When the earth is at A, the vernal equinox, the line of intersection of the ecliptic and equator, passes through the sun S, and the light just reaches the poles *n* and *s;* so that, as the earth rotates on *ns*, every place is one-half of the time in the light, and the other half in darkness. The days and nights are therefore equal.

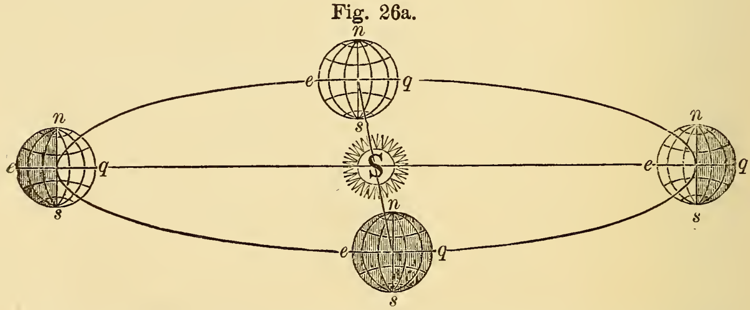
As the earth passes on toward B, the light reaches beyond

the north pole more and more, till at B, the summer solstice, it extends 23½° beyond *n*, and falls as much short of *s*, the sun being now north of the equator *eq*. As the earth now rotates on *ns*, all places north of *eq* are in the light longer than in the shade, and the reverse is true of all places south of *eq*. It is summer in the northern hemisphere, and winter in the southern.

On the 22d of September, the earth arrives at C, the autumnal equinox; the intersection of the two planes again passes through the sun, the light once more reaches the poles, and the days and nights are equal.

At D, the winter solstice, the north pole *n* is turned as far as possible into the shade, and *s* into the light. Every place north of *eq* is in the light a shorter time than in the darkness, and the reverse south of *eq*. It is now winter in the northern hemisphere, and summer in the southern.

Fig. 26a.



If the equator were in the same plane with the ecliptic, the case would be represented by Fig. 26a. The axis *ns* would then be perpendicular to the ecliptic as well as to the equator, the circle of illumination would always reach just to the poles *n* and *s*, and in the daily rotation, every place would be half the time in the sunlight, and half in the darkness. There would, therefore, be no inequality of day and night, and no change of seasons.

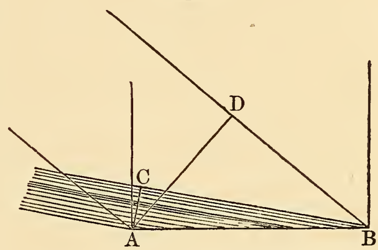
67. *Causes of heat in summer and cold in winter.*—These are *two*.

1st. The length of the *day* compared with the night. The heat of the earth is passing off by radiation during the whole time, whether the sun shines or not But the earth receives

heat from the sun, only while the sun is above the horizon. Hence, the longer the period of sunshine, compared with the time of a diurnal revolution, the greater the heat. For this reason, therefore, the summer is warmer than the winter.

2d. The different inclination of the rays to the general surface of the earth. The number of rays falling on a given surface, varies as the *sine of inclination.* Let AB (Fig. 27) be the breadth of the surface. If the rays fall on it at the angle ABC, the perpendicular breadth of the beam is AC; if at the angle ABD, the breadth of the beam is AD; while, if they fall perpendicularly, the breadth of the beam is AB itself. Now, the number of rays in the beam obviously varies as its perpendicular breadth. But these breadths, AC, AD, and AB, are as the sines of the several inclinations. In summer, the sun rises to a greater elevation each day than at other seasons, and therefore sheds a greater quantity of heat on that part of the earth.

Fig. 27.



Ex. 1. What is the relative quantity of direct heat from the sun at noon, on two equal horizontal areas, one in latitude 75° N., the other 30° N., when the sun's declination is 19° N.?

Ans. As 100 : 175½.

Ex. 2. Find the ratio, as in Ex. 1, in latitude 50° N. and latitude 45° S., when the sun's declination is 15° 45′ S.

Ans. As 100 : 212.4.

68. *Why the greatest heat is later than the summer solstice, and the greatest cold later than the winter solstice.*—If the sun sheds on a given surface more heat each day than the surface loses by radiation, then the heat accumulates from day to day. This is the case during the long days of summer; and more heat is gained than lost, till a month or more after the summer solstice. For a like reason, during the middle hours of the day, heat is received from the sun more rapidly than it is lost by radiation, so that the hottest hour is 2 or 3 o'clock P. M.

In the winter, on the contrary, the loss by radiation exceeds the quantity received from the sun, during all the shortest days, so that the temperature descends till many weeks after the winter solstice.

If loss by radiation were at a uniform rate at all temperatures, and the temperature of successive years should remain constant, as it now is, then the greatest heat would be near the autumnal equinox, and the greatest cold near the vernal equinox, the times when the surface receives heat at the mean rate.

On the contrary, if the existing amount of loss by radiation were distributed so as to be exactly proportional to the accessions received from the sun, there would be *no* change of temperature at the different seasons of the year or the different hours of the day.

But the radiation of heat follows neither of these laws; the quantity radiated is greater, when the quantity received is greater, but it does not vary at so rapid a rate.

69. *No change of seasons, if there were no obliquity.*—The angle between the planes of the two motions of the earth being the cause of the change of seasons, it follows that there would be no such change if those motions were in the same plane. If, while the earth advances in its orbit about the sun, it should rotate in the same direction on its axis, then the sun would always be in the plane of the equator, and would, every day, describe the equator as its diurnal circle, rising exactly in the east, culminating at a zenith distance equal to the latitude of the place, and setting exactly in the west. At the equator, the sun would always follow the prime vertical, and at either pole it would always be passing round in the horizon. See Fig. 26a.

70. *The greatest changes of season, if the obliquity were 90°.*—If, while the earth revolves on its axis from west to east, it should pass around the sun in a plane lying north and south, then the ecliptic would pass through the north and south poles, and the solstices would be at the poles. Hence, at a station on the equator, the sun would, during the year, describe the prime vertical and various small circles parallel to it, down to the north and south points of the horizon, where it would be

stationary alternately at the times of the solstices. At the equator, therefore, there would be an alternation from summer to winter, or the reverse, every three months.

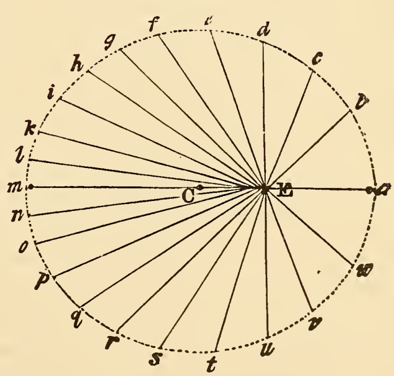
At either pole there would be but one summer and one winter in a year; but the extremes would be far more intense. For the sun, in describing diurnal circles parallel to the horizon, would occupy six months in ascending to the zenith and returning to the horizon, and the remaining six months in performing corresponding revolutions below the horizon.

At intermediate places, the extremes of the seasons would also be intermediate.

71. *Mode of determining the form of the earth's orbit.*—The earth's orbit is an ellipse described about the sun, which is situated in one of its foci. This is ascertained by observing the changes in the sun's apparent diameter throughout the year. When the sun appears smallest, it is most distant; and when largest, it is nearest. And its distance, in all cases, varies inversely as its apparent diameter. Therefore, if the sun's angular diameter be accurately measured as frequently as possible, the reciprocals of those angles express the relative distances; and these distances determine the form of the orbit.

Thus, suppose the earth to be at E (Fig. 28), and that the sun's apparent diameter is measured when in the direction E*a*. After it has advanced eastward some days, so as to be seen in the direction E*b*, let another measurement be made; and so on, at every opportunity through the year. Then let E*a*, E*b*, E*c*, etc., be made proportional to the reciprocals of the apparent diameters, and be laid down at angles equal to the angular changes of the sun's place. A line, *a b m v*, passing through their extremities, shows the form of the sun's apparent orbit

Fig. 28.



about the earth, and therefore the form of the earth's real orbit about the sun.

In this manner, even while ignorant of the *size* of the orbit, we learn that its *form* is an ellipse, and that the sun occupies one of its foci.

72. *Definitions relating to a planetary orbit.*—Let E be the focus occupied by the sun, and *am* the major axis of an elliptical orbit described about it; the nearest point, *a*, is called the *perihelion*, and the most distant point, *m*, the *aphelion*. The two points *a* and *m* are also called the *apsides*. The point *a* is sometimes called the *lower apsis*, and *m* the *higher apsis*. The varying distance, E*a*, E*b*, E*h*, etc., is called the *radius vector*. If the major axis, *am*, is bisected in C, the ratio of EC to the semi-major axis, *a*C, is called the *eccentricity* of the orbit. The less EC is, compared with *a*C, the less is the eccentricity, and the nearer does the ellipse approach to a circle. If E coincides with C, the eccentricity is nothing, and the orbit is a circle.

73. *The earth's orbit very nearly circular.*—The eccentricity of the earth's orbit in 1850 was 0.01677, and is very slowly diminishing. This fraction is about $\frac{1}{60}$,—that is, EC (Fig. 28) is $\frac{1}{60}$ of *a*C. As *a*C, in this figure, is about one inch long, EC should be only $\frac{1}{60}$ of an inch, in order to represent correctly the proportions of the earth's orbit. If it were thus drawn, it could not be distinguished from a circle in its appearance; for the minor axis, as may be easily computed, would be shorter than the major axis by only $\frac{3}{10000}$ of an inch.

74. *Position of the line of apsides.*—The direction of the major axis of the earth's orbit, or the line of apsides, is slowly changing; but at present it passes through the 10th degree of Cancer and Capricorn, as represented in Fig. 25. The earth is at perihelion on the 1st of January, and at aphelion on the 1st of July. We are therefore nearest to the sun in the winter of the northern hemisphere, and furthest from it in the summer.

75. *Distance from the sun, as affecting the seasons.*—The

intensity of the sun's heat at the earth, as well as that of its light, varies inversely as the square of our distance from it. On this account, the intensity of heat at perihelion is to that at aphelion as $61^2 : 59^2$, which is nearly as 31 : 29. Therefore, so far as distance is concerned, the earth receives $\frac{1}{15}$ more heat on the 1st of January than on the 1st of July. This produces a slight effect to mitigate the severity of cold in winter and o. heat in summer, in the northern hemisphere, and to aggravate the same in the southern hemisphere. But, on account of changes going on in the places of the equinoxes and apsides, this modifying effect will be reversed after the lapse of about 10,000 years.

CHAPTER V.

SIDEREAL TIME.—MEAN AND APPARENT SOLAR TIME.—THE CALENDAR.

76. *The sidereal day.*—This is the interval of time which elapses between two successive culminations of a star (Art. 43). The length of this interval appears to be invariable, whatever star is observed, or in whatever season or year the observation is made. On this account, the sidereal day is regarded as the true period of the earth's rotation on its axis. In order to reckon by sidereal time, the moment chosen for the beginning of each sidereal day is the moment when the vernal equinox culminates. The sidereal clock, if correct, then points to 0h. 0m. 0s. Each sidereal day is divided into 24 sidereal hours, each hour into 60 sidereal minutes, and each minute into 60 sidereal seconds.

77. *The mean solar day.*—This is the mean interval between two successive culminations of the sun. It will be shown presently, that these intervals vary throughout the year. As the sun, by the annual motion, is advancing eastward continually among the stars, the solar day must always be longer than the

sidereal day. For, if the sun and a star were on the meridian of a place together, then, while that place passes around eastward till its meridian meets the star again, the sun has advanced eastward nearly a degree, and the place must revolve nearly a degree more than one revolution before its meridian will reach the sun. This will require nearly 4 minutes of time; for, in the diurnal motion, 15° correspond to one hour, and therefore 1° to $\frac{1}{15}$ of an hour—that is, four minutes.

78. *The relation of sidereal time to mean solar time.*—As the sun, in its apparent annual motion, describes 360° in 365.24 days, it will, in one day, on an average, pass over $360° \div 365.24 = 59' \ 8.35''$, or nearly 1°, as before stated. But, by the diurnal motion, a given place on the earth in one solar day describes 360° plus the above arc. Therefore, $360° \ 59' \ 8.35'' : 59' \ 8.35'' :: 24h. : 3m. \ 55.9s.$ of solar time. This is the excess of the mean solar day above a sidereal day. And one sidereal hour, minute, or second is to one solar hour, minute, or second as $360° : 360° \ 59' \ 8.35''$,—that is, as $1 : 1.0027379$. Therefore, to reduce a given period of time from the mean solar to the sidereal reckoning, multiply by 1.0027379; and to reduce sidereal time to mean solar time, divide by the same number.

79. *The apparent solar day.*—This is the *actual* interval between two successive culminations of the sun. And this interval changes its length from day to day through the entire year, being sometimes greater, and sometimes less than the mean solar day.

In keeping solar time by clocks and watches, it is customary, for convenience, to aim to keep the mean rather than the apparent time, and to regard the sun as going alternately too fast and too slow.

80. *First cause of inequality in apparent solar days.*—One cause of inequality of days, as measured by the sun, is found in the elliptical form of the earth's orbit, and the consequent unequal increments of longitude made by the sun from day to day. At P (Fig. 25) the sun is nearest to us, and at A it is most distant. The motion in the parts of the orbit near P

would therefore *appear* greater than in the parts near A, even if it were uniform in all parts. But, besides this, as will be shown in Chapter VIII, the motion is *really* greatest at P and least at A. For both these reasons, then, the sun, while in the nearer half of its orbit, passes over the longest arcs each day in the ecliptic—that is to say, in longitude—and the shortest arcs, in the half most distant from us. The sun, in fact, occupies nearly 8 days more time in describing the remote half than the nearer one.

Recollecting, now, that a solar day consists of a sidereal day, plus the time of describing diurnally the arc which the sun, in the mean time, advances annually, it is clear that if this daily arc is longer, the solar day is longer; and if shorter, the solar day is shorter.

So far as this cause is concerned, therefore, the longest solar day would be the 1st of January, and the shortest, the 1st of July; and about half-way from P to A, and from A to P, the apparent days would have their mean length.

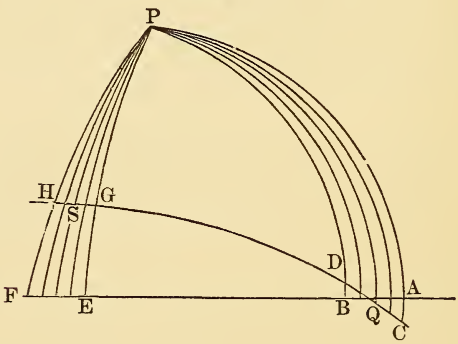
81. *Second cause of inequality in apparent solar days.*—But the solar days are unequal for another reason—the obliquity of the ecliptic to the equator. Time is measured by arcs of the equator. But the sun's daily advance toward the east is made in the ecliptic. Even if the daily increments of the sun's *longitude* were equal, those of its *right ascension* would be unequal, and therefore the solar days unequal.

Let Fig. 29 represent a portion of the celestial sphere, AF a part of the equator projected in a straight line, CH a corresponding part of the ecliptic, Q the vernal equinox, S the summer solstice, and P the north pole. Draw through P a few meridians, dividing that part of the ecliptic near Q into short arcs, to represent the daily increments of the sun's longitude on CH, and of its right ascension on AF. These meridians are oblique to CD, but perpendicular to AB. Hence, as AQC is a right-angled triangle, QC is longer than AQ; so also, DQ is longer than BQ; and thus each part of CD is longer than the corresponding part of AB;—that is, the increments of the sun's right ascension, near the equinoxes, are less than those of its longitude. The obliquity, therefore, by short-

ening these increments of right ascension, shortens the solar days.

But if meridians are drawn to that part of the ecliptic near S, the arcs GH and EF are about parallel to each other, and the increments on the equator are not shortened, as they are at Q. But, on the other hand, the divergency of the meridians causes EF to be longer than GH, and each part of EF longer than the corresponding part of GH. At the solstices, therefore, the increments of right ascension are lengthened by the divergency of the meridians, and hence the solar days are lengthened also. About midway between the equinox and solstice, the two effects just described neutralize each other, and the daily arcs of right ascension, so far as this cause is concerned, are at their mean value.

Fig. 29.



82. *Location of extreme and mean solar days from each cause.*—Suppose the first cause alone in operation, and that the sun and a uniform clock agree with each other at P (Fig. 25), on the 1st of January. Then, as the solar days are longer than their mean, the sun becomes slower, compared with the clock, from day to day, for about three months, when the days will have reached their mean length, at a point near half-way from P to A. Afterward, the days being diminished below the mean, the sun slowly gains on the clock, and catches up with it at A, July 1st. But the days now being shortest of all, the sun is immediately in advance of the clock, and most of all at a point half-way from A to P. The gain and loss compensate

each other from A to P, as they did from P to A. Thus mean and apparent time would agree twice in a year, at intervals of six months, if eccentricity of orbit were the only cause of irregularity.

Again, if the second cause alone existed, and we suppose the sun and clock to agree at the equinox Q (Fig. 29), then the sun gains on the clock every day, on account of the short arcs of right ascension near Q. In about $1\frac{1}{2}$ months, however, the days reach their mean length, the sun begins to lose what it has gained, and at S, June 21st, the sun and clock are again together. But the sun is now losing, falls behind the clock, and is furthest behind midway between the solstice and the next equinox. The autumnal equinox and the winter solstice are, in like manner, points of time at which the clock and sun agree with each other. Thus, if the second were the only cause of irregularity, the mean and apparent time would agree four times in a year, at intervals of about three months each.

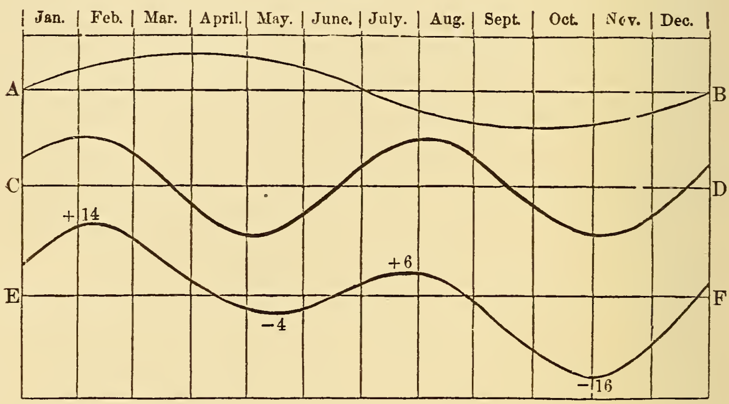
83. *The equation of time.*—The difference between mean time and apparent time, on any given day, is the equation of time for that day. If the sun is slow, the equation must be added to the apparent time; if fast, it must be subtracted from it, in order to give mean time.

We have seen by the two preceding articles that, on account of eccentricity of orbit, the equation would be reduced to zero twice in a year; and, on account of obliquity of ecliptic and equator, it would be zero four times in a year. The joint effect of these two causes is, to reduce the equation to zero four times in a year, at *unequal* intervals of time.

84. *The equation of time represented graphically.*—The ordinates of the curves in Fig. 30 exhibit to the eye the equation of time as depending on each cause by itself, and on the two conjointly. The relative lengths of the ordinates above and below AB show the positive and negative equations, as caused by eccentricity, and those on CD the equations as caused by obliquity; while the algebraic sum of these on each vertical line, gives the resultant effect on the line EF. The figure shows that the equation reaches its first maximum, + 14 minutes, on the 11th of February; its first minimum, — 4

minutes, May 14th; its second maximum, + 6 minutes, July 26th; and its second minimum, — 16 minutes, November 2d. The four times of agreement, when the equation is zero, are shown by the intersections; they occur April 15th, June 15th, September 1st, and December 24th. The sign + shows that the sun is on the meridian *after* mean noon, the sign — *before* mean noon.

Fig. 30.



85. *Civil and astronomical time.*—The mean solar day, when employed for *civil* purposes, is supposed to begin and end at midnight, and is divided into hours, numbering from 1 to 12 A. M., and then from 1 to 12 P. M. But the *astronomical* day (which is also the mean solar day) begins and ends at noon, 12 hours later than the corresponding civil day, and its hours are counted from 1 to 24. Thus, the astronomical date, April 12d. 20h., is the same as the civil date, April 13th, 8 o'clock A. M.

86. *The Julian calendar.*—The period in which the sun passes from the vernal equinox to the same point again, is called the *tropical year.* In that period the round of the seasons is exactly completed. The length of the tropical year is 365d. 5h. 48m. 46.15s. This is so near 365¼ days, that in the adjustment of the calendar made by Julius Cæsar (hence called the *Julian* calendar), three successive years were made to contain 365 days each, and the fourth 366 days. The additional day is called the *intercalary* day. In this calendar it was introduced by reckoning twice the 6th day before the

Kalends of March; and hence the year containing this additional day was called the *bissextile*. The intercalary day is now the 29th of February, and the year containing such a day is called *leap-year*.

87. *The Gregorian calendar.*—By calling the tropical year $365\frac{1}{4}$ days, the Julian calendar makes it more than 11 minutes too long, and the intercalation of one day in four years is therefore too great. This excess amounts to more than 18 hours in a century. Hence, by dropping the intercalary day three times in four centuries, the adjustment is nearly complete. The Julian calendar, thus amended, is called the *Gregorian* calendar, because adopted under Pope Gregory XIII. At that time, 1582, the vernal equinox, by the error of the Julian calendar, had fallen back to March 11th. To bring the equinox to its proper date, 10 days were first dropped (the 5th being called the 15th), and then the following system was adopted.

Every year, not exactly divisible by 4, has 365 days.

Every year, divisible by 4, and not by 100, has 366 days.

Every year, divisible by 100, and not by 400, has 365 days.

Every year, divisible by 400, has 366 days.

The Gregorian calendar will not be correct perpetually, but the error will not amount to a day in 4,000 years.

The nation of Russia has not yet adopted the Gregorian calendar, so that there is now a discrepancy of 12 days between their dates and those of other nations. The reckoning still used by them is known as *old style*, and is distinguished by appending the letters O. S. to every date.

88. *How to compare days of the month and of the week in passing from one year to another.*—A common year of 365 days contains 52 weeks and *one* day; a leap-year contains 52 weeks and *two* days. Hence, a year usually begins a day later in the week than the year previous. And, generally, any day of any month is one day later in the week than the same day of the preceding year. Thus, July 4th, 1884, falls on Friday; 1885, on Saturday; 1886, on Sunday. But, in leap year, this rule applies only till the end of February. From that time to the same date in the year following, every day of a

month falls *two* days later in the week than in the previous year. Thus, July 4th, 1883, is Wednesday; 1884, Friday. And February 2d, 1884, is Saturday; 1885, it is Monday.

Table I., at the end of the volume, contains a complete calendar for 77 centuries.

CHAPTER VI.

CURVILINEAR MOTION.—SPHEROIDAL FORM OF THE EARTH.—ITS DENSITY.—PROOFS OF ITS ROTATION ON AN AXIS.

89. *Projectile and centripetal forces.*—Motion in a curve line is always the effect of two forces; one, an *impulse* which, acting alone, would have caused a uniform motion in a straight line, and whose influence is always retained in the curve motion; the other, a *continued* force, which constantly urges the moving body toward some point out of the original line of motion. The first is called the *projectile* force, the other the *centripetal* force. If the action of the latter were to cease at any moment, the body by its inertia would from that moment continue uniformly in the direction in which it was then moving. Such motion in the tangent may be regarded as the effect of an impulse first given in the direction of that tangent. This supposed impulse is the projectile force for the moment in question; but it is in truth the resultant of the original impulse, and the infinite series of actions already produced by the centripetal force.

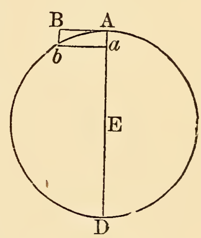
The centripetal force may be resolved into two components one in the direction of the tangent, the other perpendicular to it. The tangential component will accelerate or retard the motion in the curve according as it acts with the projectile force or in opposition to it. When the body moves in the circumference of a circle, the tangential component of the centripetal force is 0, and hence the motion is uniform.

90. *Centrifugal force.*—When a body moves in a curve,

since by its inertia it tends to proceed in the tangent at that point, there is a continual outward pressure directed *from* the center of force: this is called the *centrifugal* force. It is always opposed to the centripetal force, and in *circular* motion is always *equal* to it. It must not be viewed as a third force introduced to explain curvilinear motion, but as that component of the projectile force which acts in opposition to the centripetal force.

91. *First law of centrifugal force in circular motion.*— When a body moves in a circular path, its centrifugal (or centripetal) force *varies as the square of the velocity divided by the radius.* Let Ab (Fig. 31) $= v$, the space passed over in one second. The projectile force is then represented by AB, and the body would move in that line uniformly, were it not for the centripetal force acting toward E, and thus deflecting it into Ab. Aa being the distance through which the body falls in one second, 2Aa or c represents the centripetal force. Let AE $= r$. Then Aa : Ab : : Ab : AD; or $\frac{1}{2}c : v :: v : 2r$, and $c = \frac{v^2}{r}$. As the centripetal and centrifugal forces are equal in circular motion, c may represent either in value, though they are opposite in direction. Hence, in a given circle, where r is constant, the force either toward or from the center varies as v^2, the square of the velocity. In whirling a ball, for instance, with a string of given length, if the velocity is doubled, the strain upon the string (the centrifugal force) is four times as great, and the strength of the string (the centripetal force) needs also to be four times as great. So, if a train of cars goes round a curve with a velocity $1\frac{1}{2}$ times that which is intended, its tendency to be thrown from the track is increased $2\frac{1}{4}$ times.

Fig. 31.



92. *Second law of centrifugal force in circular motion.*— When the path of a body is circular, its centripetal or centrifugal force *varies as the radius of the circle divided by the square of the time of revolution.*

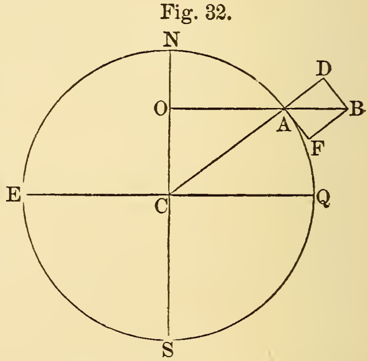
Let $t =$ the time of describing the whole circumference $2\pi r$;

and let the velocity per second $= v$. Therefore $2\pi r = vt$, and $v = \frac{2\pi r}{t}$; $\therefore v^2 = \frac{4\pi^2 r^2}{t^2}$. But (Art. 91) $c = \frac{v^2}{r} = \frac{4\pi^2 r}{t^2}$, which varies as $\frac{r}{t^2}$. Aa or $\frac{1}{2}c = \frac{2\pi^2 r}{t^2}$.

Hence, if the time of revolution is the same, the attraction to the center must be increased as the radius is increased; for then $c \propto r$. Thus, if a string is twice as long, it must have twice the strength, in order to whirl a ball at the same rate of revolution.

93. *Centrifugal force on the earth's surface.*—As the earth makes its diurnal rotation, all free particles upon it are influenced by the centrifugal force. Let NS (Fig 32) be the axis, and A a particle describing a circle with the radius AO. If AB, in the plane of that circle, represent the centrifugal force, resolve it into AD on CA produced, and AF, tangent to the meridian NQS. The effect of AD is to diminish the weight of the particle, while the effect of AF is to urge it horizontally toward the equator. If the surface, then, consists of yielding matter, as water, the spherical form can not be retained, but the parts about the poles, N and S, will be depressed, and those about the equator, EQ, will be elevated. At each point between the pole and the equator, a particle is held in equilibrium, by that component, AF, of the centrifugal force which urges it toward the equator, and that component of gravity which urges it down the inclined surface toward the pole.

Fig. 32.



94. *Loss of weight at the equator caused by rotation.*—Let the weight of a body, w, be taken to express the force of gravity, and let $\frac{1}{2}g$ ($= 16\frac{1}{12}$ feet) be the distance fallen through by this body in one second. Now, c is the force by which Aa

(Fig 31) is described in one second; and $Aa = \frac{2\pi^2 r}{t^2}$ (Art. 92)
Hence,

$$w : c :: \tfrac{1}{2}g : \frac{2\pi^2 r}{t^2};$$

$$\therefore c = w \times \frac{4\pi^2 r}{gt^2}.$$

Using the values of the letters in the fraction, we obtain c, the centrifugal force, in terms of w, the weight of the body.

The equatorial radius of the earth, r, is 3962.8 miles = 20,923,584 feet.

The earth makes one rotation in 24 sidereal hours = 86,400 sidereal seconds. Reducing this to solar seconds (Art. 78), we find

$$t = 86{,}164\text{s}. \quad \text{Hence,}$$

$$c = w \times \frac{4 \times 3.14159^2 \times 20{,}923{,}584}{32\frac{1}{6} \times 86{,}164^2} = \frac{w}{289}.$$

And, since the centrifugal force at the equator acts directly from the center, a body at the equator loses $\frac{1}{289}$ of its weight by the rotation of the earth.

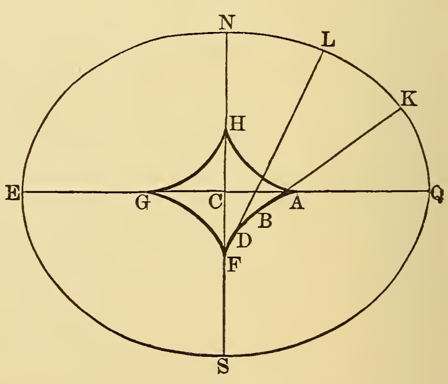
95. *Loss of weight by rotation at other latitudes.*—Since c varies as r (Art. 92), the centrifugal force is greatest at the equator, and zero at the poles, and the force at the equator is to that at any latitude A (Fig. 32) as QC : AO —that is, as rad : cos lat. But, except at the equator, the centrifugal force does not directly oppose gravity. If AB is the whole centrifugal force at A, AD is the component of it which acts against gravity. But AB : AD :: AC : AO :: rad : cos lat. So that the loss of weight is diminished again in the same ratio as before. Therefore, the loss of weight at the equator is to that at any given latitude, as rad^2 : cos^2 of latitude.

96. *Whole loss of weight at the equator.*—It is found by observations made with the pendulum, that the weight of a body at the equator is $\frac{1}{194}$ less than that at the poles. But the

loss from centrifugal force is only $\frac{1}{289}$. Subtracting this from $\frac{1}{194}$, the remainder is very nearly $\frac{1}{590}$, a loss of weight at the equator which must be ascribed to some other cause. This cause is the oblateness itself, by which the equator is more distant from the center than the poles are.

97. *Spheroidal form of the earth found by measurement.*—Not only is the oblate form of the earth inferred from its rotation on its axis, but the measurement of the length of a degree of latitude, at various distances from the equator, proves that the meridians of the earth are ellipses, whose major axes are in the plane of the equator, and their common minor axis a line joining the poles. If the meridians were circles, all the degrees of latitude would be of the same absolute length, but it has been ascertained, by numerous and most accurate trigometrical surveys, that the length of a degree of latitude is least at the equator, and increases toward the poles. But if the degree lengthens as we go toward the pole, then the radius must lengthen in the same proportion, and therefore the curve, belonging to a larger circle, must become more flattened. And this change of curvature belongs to an ellipse, not to a circle. Thus, at Q (Fig. 33) the degree is shortest, longer at K, still longer at L, and so on to the pole. The center of the arc Q is at A, nearer than the center of the earth; the center of K is B, of L is D, and of the polar arc it is F, beyond the center C. Thus, the centers of curvature of the elliptical quadrant QN lie on the curve ABDF, which is the evolute of that quadrant. Each meridian quadrant is in like manner the involute of a curve, and their four evolutes form the figure AFGH about the center. No part of a meridian has its center of curvature at the center of the earth.

Fig. 33.



The following numbers express both the size and the form of the earth:

Equatorial diameter . .	7925.604 miles.
Polar diameter . . .	7899.100 "
Mean diameter . . .	7912.357 "
Difference of diameters . .	26.504 "

The difference of diameters is $\frac{1}{299}$ of the equatorial diameter; this is called the compression of the poles, or the *ellipticity* of the earth.

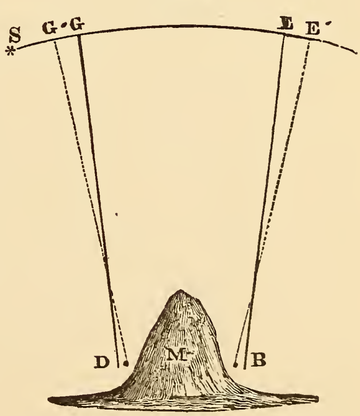
So slight is the oblateness above described, that an exact model of the earth could not be distinguished by sight or touch from a perfect sphere.

The volume of the earth $= (7912.357)^3 \times \frac{\pi}{6} = 259{,}400{,}000{,}000$ cubic miles.

98. *The equatorial belt.*—If we imagine a sphere constructed on the polar diameter of the earth, the difference between the sphere and spheroid will be a sort of shell or ring, thirteen miles thick at the equator, and growing thinner on every side to the poles. This is sometimes called the equatorial ring or belt of the earth, and it produces sensible effects on the earth's relations to the moon and sun.

99. *Weight and density of the earth.*—The earth's mass, and therefore its density, can be obtained by comparing the effects produced upon a plumb-line, by the earth and a mountain of known weight. Let M (Fig. 34) be an abrupt mountain situated alone on a plain, and let a station, B, be selected on the north side of it, and another, D, in the same meridian, on its south side, for measuring the zenith distances of stars. If

Fig. 34.



the mountain were not present, the plumb-line of the zenith sector would hang in the lines B and D, and would mark E and G as the zeniths of the stations. But the attraction of the mountain draws the plumb-line toward it, so as to point to the false zeniths E′ and G′. When the star S, therefore, culminates, its apparent zenith distance, SE′, is measured at one station, and at another culmination, SG′ is measured. The difference, SE′ − SG′, is the distance between the apparent zeniths. The distance, EG, between the true zeniths, is the same as the difference of latitude between the stations B and D. Let a trigonometrical survey, therefore, be made around the mountain, and thus the arc BD, or its equal EG, be found. E′G′ − EG = the sum of the two angles by which the plumb-line is drawn from a vertical position at the two stations. The volume and density of the mountain being measured, and the angle being found, as above, by which it draws a plumb-line from a true vertical, we have the means of determining the mass of the earth. And, as its volume is known, its density is inferred. Observations of this kind were made near Mount Schehallien, Scotland, by Dr. Maskelyne, who found the deviation of the plumb-line to be a little more than 6″.

The mean density of the earth, as deduced from a great number of results, obtained by this and other methods, is 5.46,—that is, the earth, as a whole, is 5.46 times the weight of the same volume of water. Calling the weight of a cubic foot of water 62½ lbs., the weight of the earth is somewhat more than 6,000,000,000,000,000,000,000 tons.

100. *Proofs of the earth's diurnal rotation.*

1. To suppose the earth to rotate eastward on its axis, is the only reasonable way of explaining the fact, that all the millions of fixed stars, at various and immense distances from us, in large and in small circles of the sphere, perform their apparent revolutions about us in precisely the *same length of time* —viz., one sidereal day.

2. Without supposing the earth to rotate on its axis, we can not account for the *oblate form* of the waters of the ocean. Whatever form the solid parts might have, the movable portion would be spherical, if the earth were at rest. Moreover, the

degree of oblateness is exactly that which is required on a sphere having the diameter and mass of the earth, if it be supposed to rotate once in 24 hours.

3. The weight of a body at the equator, compared with that at the poles, is too small to be wholly accounted for by increased distance. *Centrifugal force*, arising from rotation, can alone explain the remaining difference.

4. A body dropped from a great height strikes *further east* than the vertical line in which it began to fall. If the earth rotates, the top of a tower moves faster than the base; and therefore a body let fall from the top, retaining the eastward motion of that point, will strike further east than the base. At the equator, this distance would be near 2 inches, for a fall of 500 feet. Numerous experiments on the fall of bodies through great distances have been very carefully made by different individuals, and in different latitudes. And they all concur in proving that a body in falling deviates from a vertical line toward the east.

5. It is proved by the *vibrations of a pendulum* that the earth rotates eastward. Let us suppose a weight to be suspended by a long fine wire, and then made to vibrate in a plane. The plane in which the wire and weight move is vertical, and passes through the point of suspension. The weight itself may be considered as describing a straight horizontal line. On account of inertia, the weight tends to keep always in the same line, or (if the point of suspension be moved) in a line parallel to itself. And it will always remain strictly parallel to itself, provided it can at the same time remain horizontal, and in a vertical plane passing through the point of suspension.

Thus, if at the equator the weight be made to vibrate north and south—that is, in the plane of a meridian—it will continue to do so without deviation, as the earth rotates eastward, because it will thus remain moving horizontally in a plane which passes through the point of suspension, though that plane is continually changing. In this case, the lines in which the weight vibrates are all parallel among themselves.

If the experiment be tried at the pole, and the weight be made to vibrate in the plane of a certain meridian, the point of suspension does not move *from* its place, but only revolves *in*

it; and while the earth revolves 15° per hour, the weight preserving its own plane of vibration, will *seem* to shift that plane 15° per hour in the contrary direction, keeping pace with the stars in their diurnal motion.

At localities between the equator and the pole, the line of vibration remaining horizontal, and in a vertical plane which passes through the point of suspension, can not at the same time preserve its parallelism. But it will come as near fulfilling this condition as possible. Its north extremity will deviate eastward from the meridian more or less, according as it is nearer the pole or the equator. It is proved that the deviation per hour is to 15° as the sine of latitude to radius.

When experiments are performed with sufficient care, it is found that the pendulum actually deviates eastward from the meridian, and at a rate corresponding well with the calculated result. The pendulum thus furnishes evidence that the earth rotates on its axis.

The above is known as Foucault's experiment.

6. It will be seen hereafter that the motion of the equinoctial points toward the west, called the *precession of the equinoxes*, affords an independent proof of the earth's diurnal motion.

CHAPTER VII.

THE SUN.—SOLAR SPOTS.—CONDITION OF THE SUN'S SURFACE.—THE ZODIACAL LIGHT.

101. *The form of the sun.*—The disk of the sun is always circular. And, as it presents all sides toward us in its rotation, we infer that its form must be *spherical.* But since it rotates on an axis, and its surface is in a fluid state, it might be expected to reveal a spheroidal form. The reasons why it does not are, that the force of gravity on the sun is very great, and, in consequence of the slowness of its rotation, the centrifugal force is small. It appears by calculation that the angle subtended by the equatorial and the polar diameters can not differ

from each other, except by a small fraction of a second. Its oblateness is, therefore, too slight to be perceived.

102. *Distance of the sun, and size of the earth's orbit.*—The sun's horizontal parallax is 8.″848. Therefore, the distance of the sun from the earth is found (Fig. 4) by the proportion,

$$\sin 8.''848 : \text{rad} :: 3962.802 : 92{,}381{,}000;$$

which is the distance in miles from the earth to the sun.

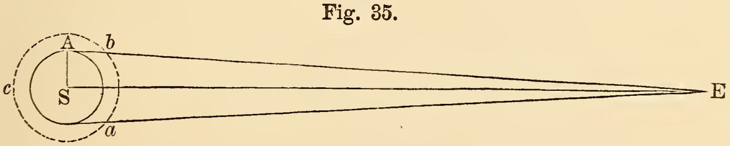
The circumference of the earth's orbit, or the distance traveled by the earth each year, is

$$92{,}381{,}000 \times 2\pi = 580{,}447{,}000 \text{ miles.}$$

103. *Velocity of the earth on its axis and in its orbit compared.*—In the diurnal motion, a place on the equator describes nearly 25,000 miles in 24 hours—that is, more than 1,000 miles per hour, or about 17 miles in a minute. In the annual motion, the earth describes 580,447,000 miles in 365¼ days, thus passing over a distance of 1,589,000 miles each day; which is about 1,103 miles in a minute, or 18.393 miles in a second. The earth's velocity in its orbit is about 65 times as great as that of the equator in the diurnal motion.

104. *To find the dimensions of the sun.*—The angle subtended by the sun's diameter may be measured by instruments. Let AES (Fig. 35) equal one-half the measured angle. Then we have rad : sin AES :: ES : AS, the semi-diameter of the sun. As the sun's mean apparent semi-diameter is 16′ 2″, and ES is 92,381,000 miles, we find the sun's radius near 430,855, and therefore its diameter 861,710 miles.

Fig. 35.



The sun's diameter is about 109 times that of the earth. And, since spheres vary as the cubes of their diameters, the volume of the sun to that of the earth is as

$$109^3 : 1^3 :: 1{,}295{,}000 : 1, \text{ nearly.}$$

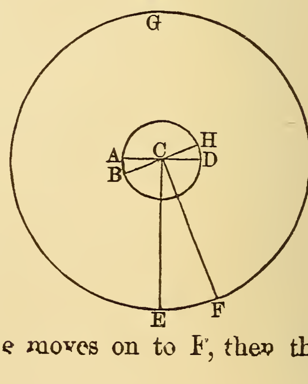
105. *The sun's mass and density.*—It is found, by methods to be described hereafter, that the sun does not exceed the earth in *mass* nearly so much as it does in *volume.* While the volumes are as 1,295,000 : 1, the masses are about as 326,800 : 1.

The density of the sun, therefore, is to that of the earth as 326,800 : 1,295,000 : : 1 : 4, nearly.

106. *Force of gravity at the surface of the sun.*—When the relative masses and diameters of bodies are known, it is easy to find the relative force of gravity on their surfaces. For $G \propto \frac{Q}{D^2}$ (Nat. Phil., Art. 16), where G represents gravity, Q the mass of the body, and D its semi-diameter. Let W represent weight at the earth, and W′ at the sun, and we have $W : W' :: \frac{1}{1^2} : \frac{326,800}{109^2} :: 1 : 27.5$. Hence, the weight of a body at the sun is 27.5 times as great as at the earth, and a body would fall 442 feet in the first second of its descent.

107. *Diurnal rotation of the sun.*—By observations on the solar spots, it is found that the sun rotates on its axis nearly in the same direction in which the earth revolves about the sun. In general, a spot which appears on the edge of the disk passes across, then disappears, and afterward reappears in the same place as at first in $27\frac{1}{4}$ days. If the earth were at rest, this would be the period of the sun's rotation on its axis. But, as the earth revolves in nearly the same direction in its orbit, the apparent rotation of the sun is longer than its real rotation. In Fig. 36, suppose the earth to be stationary at E, and that a spot on the sun appears on the disk at A. Then, after passing through B, D, H, it will appear again at A, at the end of one revolution. But, if the earth in the mean time moves on to F, then the

Fig. 36.



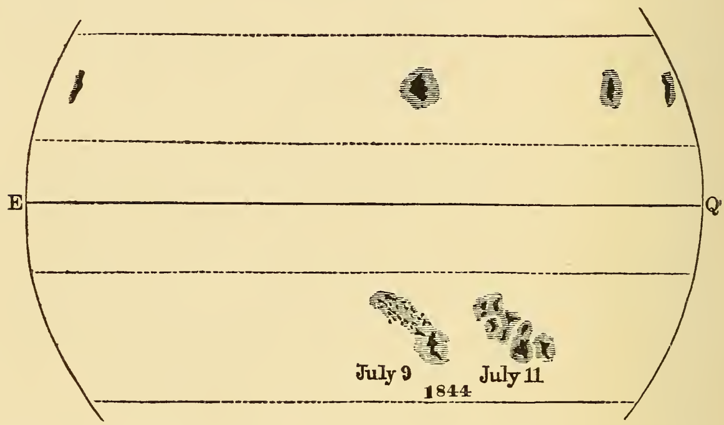
spot must pass over AB, in addition to one revolution, before it will be seen on the edge of the disk. As EC is perpendicular to AD, and FC to BH, the corresponding arcs on the two circles are obviously similar. Therefore, EGE + EF : EGE :: ADA + AB : ADA. Instead of the arcs, we may use the times of describing them; and then we have 1 year + $27\frac{1}{4}$ days : 1 year :: $27\frac{1}{4}$ days : 25 days, $8\frac{1}{2}$ hours, which is the period of the sun's rotation. Appendix A.

108. *Position of the sun's equator.*—If the solar spots always described their paths across the disk in apparent straight lines, it would be inferred that the sun's equator coincides with the plane of the ecliptic. But these lines appear straight only twice in the year, near the middle of June and of December. At other times, they appear as semi-ellipses, having the greatest breadth in March and September. The earth, therefore, passes the plane of the sun's equator in June and December. The inclination of the sun's equator to the plane of the ecliptic is found to be about $7\frac{1}{4}°$.

109.—*Appearance of the solar spots.*—On examining the sun's disk with a telescope, there is usually seen a greater or a less number of dark spots, differing from each other in form and size, and each spot generally consisting of two distinct parts, called the *macula*, or *nucleus*, and the *umbra*. The macula is black, of irregular form, and commonly surrounded by the umbra, which has a lighter shade. The two parts of the spot do not often shade into each other, but are each marked by a sharp, though irregular outline. If watched from day to day, they are seen not only to move slowly across the disk, as already stated, but they change their form and general appearance. A large spot sometimes divides into two or more smaller ones; and again a group unites into a single large spot. Sometimes a spot diminishes and disappears, first the macula, then the umbra. The reverse also happens—a spot is seen in the midst of the disk, where there was none the day before. Though only a few are commonly in sight at once, yet they have been, in some instances, counted by tens and even hundreds. Very rarely a spot is so large as to be seen by the naked eye.

Figure 37 (lower part) shows two views of the same group as seen July 9th and 11th, 1844.

Fig. 37.



110. *The spots are at the surface, and limited to a northern and a southern zone.*—Each spot appears on the disk during one-half the time of its entire revolution. It must, therefore, be *at* the surface, and not at any distance from it. For, if it revolved at any distance from the surface, as in the orbit *abc* (Fig. 35), then it would be seen on the disk only from *a* to *b*, which is less than half its orbit.

But the spots do not pass across all portions of the disk; their paths are limited to a zone which extends not more than 35° on each side of the equator; and with very few exceptions, they lie in the outer, rather than the central parts of this zone. Spots are very rarely seen within the zone lying between 10° of north and south latitude; and still more rarely in the polar zones above latitudes 35° north and 35° south. The *macular zones*, as they are sometimes called, are represented in Figure 37, limited by the dotted lines, EQ being the equator.

111. *Relation of the spots to the surface level.*—If the spots were flat surfaces on the same level with the general surface of the sun, then all their parts would be foreshortened alike,

when near the edges of the disk. If they were elevated objects, as mountains, rising above the solar atmosphere, then the umbra nearest the edge of the disk would be hidden by the darker part, and *on* the edge the spot would appear as a protuberance.

But it is proved, by multiplied observations, that the spots must be *depressions* below the general surface, and the macula a deeper depression than the umbra. For, as a spot approaches the edge, while it is foreshortened by perspective, the umbra furthest from the edge disappears first, and then the macula itself, while that part of the umbra nearest the edge is still in sight. As a spot comes from the edge toward the central part of the disk, the order of appearances is reversed. These changes are indicated in Fig. 37, upper zone. Appendix B.

112. *The general surface.*—The luminous part of the sun's surface is not uniform, nor at rest. Every portion of it is minutely mottled by spots and streaks of unequal illumination. These are called *faculæ.* And continued observation shows that these faint inequalities are also undergoing incessant changes. The faculæ are most strongly marked, and indicate the greatest agitation of surface, where a spot is about to appear, or where one has recently disappeared. Appendix C.

113. *The received theory.*—No theory so well explains the telescopic appearances of the sun, as that which in substance was proposed by Sir William Herschel, in 1801. Whatever may be the condition of the central mass, the external surface, called the *photosphere,* consists of gas in an incandescent state, while below it, within the solar atmosphere, is a cloudy stratum, less luminous than the outer surface. Whenever, from any cause, a rent is made in the photosphere, the less luminous stratum below is seen through it, as the umbra of a spot; and a smaller rent in the lower stratum reveals the denser and darker part of the sun, as the macula of the same spot. The strata in which the rents occur are in a gaseous condition; for the constant motions going on in the outlines of the spots, forbid the supposition that they consist of solid matter; and the extreme rapidity of these motions, often more

than 1,000 miles per day, is inconsistent with the idea that they are liquid.

114. *The body of the sun not necessarily dark.*—The very dark appearance of the macula may be due to its strong contrast with the intense illumination of the general surface. For it is found by experiment that the brightest artificial light which has been produced, if placed between the eye and the sun, appears as a dark spot compared with the solar surface.

115. *Cause of the spots.*—Sir John Herschel has suggested that there are reasons for considering the equatorial regions of the sun to be more heated than the other portions, so that there are currents in the solar atmosphere analogous to the trade-winds on the earth. Resulting from these currents, he supposes that occasional local winds are produced, rotating on a vertical axis, and rending the atmosphere and clouds by their centrifugal force. The ruptures thus occasioned are the spots on the sun.

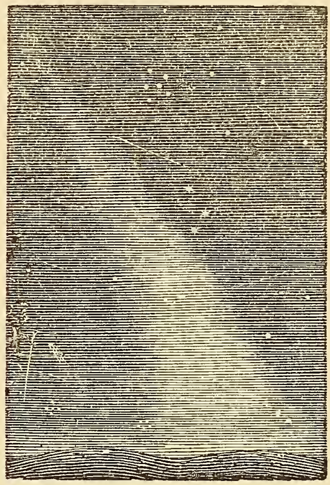
This supposition derives considerable plausibility from the considerations, that the spots are limited to narrow zones a little distance from the equator; that they sometimes differ from each other in their motions across the disk; and that, in a few instances, they have shown signs of rotation about their own centers. Appendix D.

116. *Periodicity of the spots.*—The number and size of spots vary exceedingly in different years. Sometimes for days and weeks none are to be seen; and again, for many months, the disk is never free from them. It is noticed, of late years, that their frequency alternately increases and decreases during a period of 10 or 11 years. The years in which the greatest number has been seen of late, were 1870, 1882. And those in which there were fewest, were 1867, 1878. Appendix E.

117. *The zodiacal light.*—This name is given to a faint, ill-defined light, extending along the zodiac, either in the west, after sunset, or in the east, before sunrise. It so much resembles the twilight, that it is not ordinarily noticed, because it

appears as a mere upward extension of it. It is projected on the sky as a triangle, inclined to the horizon at the same angle as the ecliptic (Fig. 38). In the evening it is best seen at the season when the ecliptic is most nearly perpendicular to the horizon, after twilight has ceased. It is therefore most conspicuous at evening in the month of February. When the air is clear, and there is no moon, it is visible till after 9 o'clock. For a like reason, the best time for seeing it before morning twilight is the month of October. The apparent extent of it, both in breadth and height, is much increased by indirect vision.

Fig. 38.



118. *Its nature.*—There has been much speculation relative to the nature of the zodiacal light. But astronomers generally regard it as a nebulosity attending the sun, and extending beyond the orbits of Mercury and Venus, and even beyond the orbit of the earth.

CHAPTER VIII.

KEPLER'S LAWS.—THE LAW OF GRAVITATION.

119. *Statement of Kepler's laws.*—From a long and laborious examination of the recorded observations of Tycho Brahe, Kepler deduced three laws relating to the movements of the planets about the sun. They are hence called Kepler's laws, and may be stated as follows.

1. *The areas described about the sun by the radius vector of an orbit, vary as the times of describing them.*

2. *The orbit of every planet is an ellipse, having the sun in one focus.*

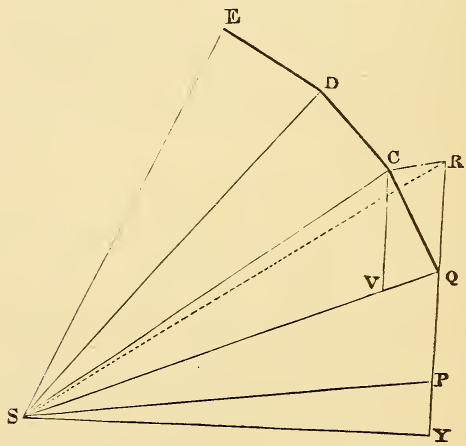
3. *The squares of the periodic times of the several planets vary as the cubes of their mean distances.*

To render the language of the third law strictly correct, the cube of the distance should be *divided by the sum of the masses of the sun and planet.* But the mass of even the largest planet is so small, compared with the sun, that the omission introduces an error which is scarcely appreciable.

Kepler established these three laws as *facts* in the solar system; but Newton afterward demonstrated, by mathematical reasoning, that they are necessarily involved in the laws of inertia and gravitation.

120. *Areas described by the radius vector.*—Whatever path a body describes under the influence of a projectile and a centripetal force, *the areas described about the center of force vary as the times of describing them.*

Fig. 39.



Let S (Fig. 39) be the center of attraction, and suppose the projectile force in the line YR to be such as to cause the body to pass over the equal spaces PQ, QR, etc., each in a certain

unit of time. When the body reaches Q, let the action toward S be sufficient to move it over QV in the same time in which by the original impulse it would describe QR. Then it will in the same time describe the diagonal QC of the parallelogram. Join RS and CS. The triangles QSC and QSR are equal; but QSR = QSP; $\therefore$ QSC = QSP,—that is, the areas described in the first and second units of time are equal. In like manner, by supposing a second action toward S to occur at C, a third at D, etc., it is proved that QCS, CDS, DES, etc., which are described in equal times, are equal. This is true, however small the unit of time between the successive actions toward S, and is therefore true when the central force acts incessantly and causes curvilinear motion. As all the areas are equal, which are described in the several units of time, therefore the areas vary as the times.

As the diagonal of each parallelogram is in the same plane with its two sides, it is obvious that the whole orbit lies in one and the same plane.

Conversely, *if areas described about a point vary as the times, the deflecting force acts toward that point.* For PSQ = QSR, as before (Fig. 39); and by supposition, PSQ = QSC; $\therefore$ QSC = QSR; hence CR is parallel to QS, and QC is the diagonal of a parallelogram, whose side QV, in which the deflecting force acts, is directed toward S.

Since it is an established fact, agreeably to Kepler's first law, that the radius vector of each planetary orbit describes areas about the sun, which vary as the times; therefore, the centripetal force, acting on the planets, is directed toward the sun.

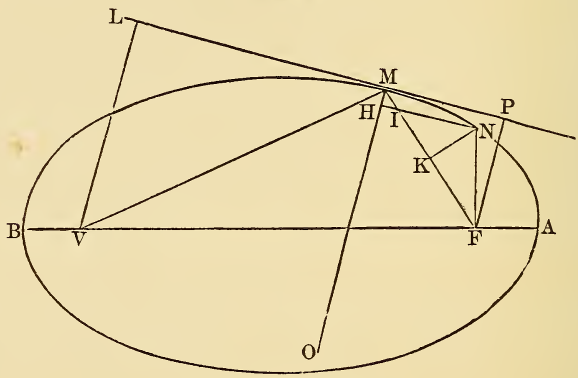
121. *The law of velocity in an orbit.*—The velocity at any point varies *inversely as the perpendicular* from the center of force to the tangent at that point.

Let SY (Fig. 39) be perpendicular to PQ; then the area $SPQ = \frac{1}{2}PQ \times SY$, which varies as $PQ \times SY$; $\therefore PQ \propto \frac{SPQ}{SY}$. But $PQ \propto V$, the velocity at P; and the area SPQ is constant; $\therefore V \propto \frac{1}{SY}$, or the velocity varies inversely as the perpendicu-

lar from S, upon the line in which the body is moving; in other words, upon the tangent of its path, if it describes a curve.

122. *Law of gravitation in an orbit, as related to distance.*—If a body describes an elliptical orbit, by a centripetal force which acts toward the focus, that *force varies inversely as the square of the distance.*

Fig. 40.



Let the body be at M (Fig. 40), and MF the radius vector at that point. Let MO be the radius of curvature at M, and therefore perpendicular to the tangent; and suppose MN to be an infinitely small arc described in a given small portion of time. Draw FP perpendicular to the tangent MP, NK to FM, and NH to MO; then PFM, MHI, KNI are similar triangles. MN, considered as a straight line, is described by the joint action of the centripetal force in the line MI, and the projectile force which is parallel to IN. The motion in MI may be regarded as uniformly accelerated, because in the infinitely small time of describing it, the centripetal force may be considered constant. Hence, 2MI may be taken as the measure of the centripetal force f (Nat. Phil., Art. 28).

Therefore, $f \propto$ MI. It is to be proved that MI $\propto \frac{1}{FM^2}$.

123. By similar triangles, MI : MH : : NI : NK;

$$\therefore \text{MI} = \text{MH}\frac{\text{NI}}{\text{NK}}.$$

Now, the chord MN is a mean proportional between the versed sine MH and the diameter 2MO; or $\text{MH} = \frac{\text{MN}^2}{2\text{MO}}$; but, as the arc is infinitely small, NH = MN; $\therefore \text{MH} = \frac{\text{NH}^2}{2\text{MO}}$. Again, the versed sine MH, and therefore HI, is infinitely small compared with NH, and NI may be substituted for NH;

$$\therefore \text{MH} = \frac{\text{NI}^2}{2\text{MO}}.$$

124. Now it is shown in conic sections, that

$$\text{MO} = \frac{p}{2}\left(\frac{\text{FM}}{\text{FP}}\right)^3;*$$

therefore, since by similar triangles $\frac{\text{FM}}{\text{FP}} = \frac{\text{NI}}{\text{NK}}$,

$$\text{MO} = \frac{p}{2}\left(\frac{\text{NI}}{\text{NK}}\right)^3.$$

Substituting this for MO in the equation for MH above, we have

$$\text{MH} = \frac{\text{NK}^3}{p\,.\,\text{NI}}.$$

Hence, in the equation for MI we have

$$\text{MI} = \frac{\text{NK}^3}{p\,.\,\text{NI}} \times \frac{\text{NI}}{\text{NK}} = \frac{1}{p}\text{NK}^2.$$

Now, the sector FMN is measured by $\frac{1}{2}$FM . NK; ∴ NK =

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* Jackson's Conic Sections. The same may be derived from Coffin's Conic Sections, Pr. V., Curvature, R^2 or $\text{MO}^2 = \frac{(\text{FM}\,.\,\text{MV})^3}{a^2b^2}$, a and b being the semi-axes; $\therefore \text{MO} = \frac{1}{ab} \times (\text{FM}\,.\,\text{MV})^{\frac{3}{2}}$. Multiply by $(b^2)^{\frac{3}{2}}$, and divide by its equal $(\text{FP}\ \ \text{VL})^{\frac{3}{2}}$; then $\text{MO} = \frac{b^2}{a}\left(\frac{\text{FM}\,.\,\text{MV}}{\text{FP}\,.\,\text{VL}}\right)^{\frac{3}{2}} = \frac{b^2}{a}\left(\frac{\text{FM}^2}{\text{FP}^2}\right)^{\frac{3}{2}}$, since FMP and VML are similar. But $\frac{b^2}{a} = \frac{p}{2}$; $\therefore \text{MO} = \frac{p}{2}\left(\frac{\text{FM}^2}{\text{FP}^2}\right)^{\frac{3}{2}} = \frac{p}{2}\left(\frac{\text{FM}}{\text{FP}}\right)^3$.

$\frac{2FMN}{FM}$; and $NK^2 = \frac{4FMN^2}{FM^2}$; $\therefore$ $MI = \frac{4FMN^2}{p \cdot FM^2}$. But as the areas described by the radius vector vary as the times, FMN is constant. Therefore,

$$MI\ (=f) \propto \frac{1}{FM^2};$$

that is, the centripetal force in the orbit varies inversely as the square of the distance.

125. *Applicable to every conic section.*—It is thus proved that, in any elliptical orbit described about the focus as the center of attraction, the intensity of that attraction varies inversely as the square of the radius vector. As there is nothing in the foregoing demonstration to limit the conclusion to the orbits which are nearly circular, like those of the planets, we are at liberty to apply it to orbits of extreme eccentricity, as those of the comets. And it is proved by Newton, in his Principia, that the same law of force is necessary, in order that a body may describe any one of the conic sections about its focus as the center of attraction.

126. *Law of gravitation as to distance, in different orbits.*—And not only does this law prevail in all parts of any one orbit, but it is true also that all the different bodies of a system, describing orbits about the same center of force, are urged toward that center by attractions which vary, from one orbit to another, inversely as the square of the distance.

Let a be the semi-major, and b the semi-minor axis of any elliptic orbit. Then a is the mean distance of all points of the orbit from the focus. By a rule of mensuration, the area of the ellipse $= \pi ab$. If $s =$ the area described by the radius vector in a unit of time, as *one second*, and $t =$ the number of seconds in the whole period of revolution, then the ellipse also $= ts$. Therefore, $\pi ab = ts$; and $t = \frac{\pi ab}{s}$; and $t^2 = \frac{\pi^2 a^2 b^2}{s^2}$. By Kepler's third law (Art. 119), $t^2 \propto a^3$; $\therefore$ $\frac{a^2 b^2}{s^2} \propto a^3$; $\therefore \frac{b^2}{a} \propto s^2$.

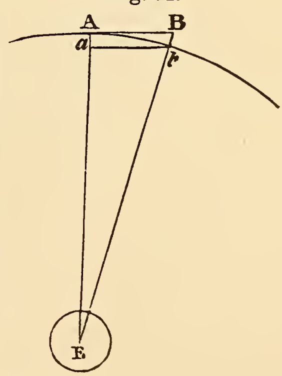
But, because the semi-parameter $\frac{p}{2}$ is a third proportional to

the semi-axes a and b, $\frac{b^2}{a}=\frac{p}{2}$; $\therefore \frac{p}{2} \propto s^2$. Hence, substituting $\frac{p}{2}$ for s^2,—that is, FMN^2,—in the equation for MI (Art. 124), we find $MI=\frac{4FMN^2}{p.FM^2}=\frac{2p}{p.FM^2}=\frac{2}{FM^2}$; $\therefore f \propto \frac{1}{FM^2}$. Or, the force varies inversely as the square of the distance, in different orbits, as well as in different parts of the same orbit.

The satellites which revolve about the planets are found to conform to Kepler's laws, and therefore the force which urges them toward their respective primaries varies in each case inversely as the square of the distance.

127. *Law of gravitation within small distances.*—But the inquiry still remains, does the law of gravity, as demonstrated in the foregoing articles, hold good at the *smallest* distances also? For example, do the tendencies of bodies resting on the earth, and of those elevated in the air, and of the moon toward the earth's center, come under the same general law? This is the very question which presented itself to the mind of Newton, after he had discovered that the force which deflects the planets from their lines of motion toward the sun, varies inversely as the square of their distance from it. As he noticed the fall of an apple, the inquiry arose, may not this *fall* be of the same nature as the *bending* of the moon's path toward the earth, and may not the force in the two cases be as the squares of the distances inversely?

Fig. 41.



The distance through which the moon actually descends in one second may be represented by Aa (Fig. 41), Ab being the arc described in the same time. For, as the moon was going toward B, it would not have deviated from the line AB, if some force had not turned it aside. This influence must be directed toward the earth, E, because it is about E that the radius is known to describe areas proportional to the times (Art. 120). There-

fore, Bb, or the versed sine Aa (which may be considered equal to it), is the distance fallen through in one second. Now, the circumference of the moon's orbit, divided by the number of seconds occupied in describing it, gives the arc Ab. This arc and its chord may be considered the same, and by geometry we have $2\,AE : Ab :: Ab : Aa = 0.0535$ of an inch.

At the surface of the earth, a body falls $16\frac{1}{12}$ feet in the first second. On the supposition that gravity varies inversely as the square of the distance, we find the fall in one second at the moon, by the proportion, the square of the moon's distance : square of the earth's radius :: $16\frac{1}{12}$ feet : 0.0536 of an inch, agreeing very accurately with the distance which the moon actually falls from a tangent in one second. Therefore, a body falling at the surface of a planet, and a satellite revolving about it, are both subject to the same law of centripetal force.

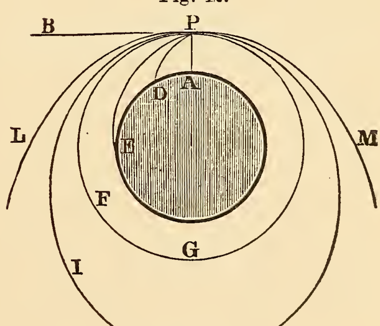
128. *The law prevails throughout the solar system.*—As will appear hereafter, there are numerous disturbances produced upon the motion of each body in the system by the attraction of every other. Every one of these disturbing influences is measured, by applying the law of distance already mentioned. If a planet or comet moves toward a planet for a certain length of time, it is accelerated; and its acceleration is greater, as the square of the distance is less; and it is retarded, according to the same law, when departing from it.

129. *The law of gravitation, as related to the quantity of matter.*—The force of gravity varies directly as the quantity of matter. In Mechanics, we infer the existence of this law from the fact that all bodies, light and heavy, and of every kind of material, fall with equal velocity toward the earth. So, in the solar system, a planet and all its satellites, when at equal distances from the sun, are urged toward it by forces proportional to their masses, or they could not maintain their mutual relations as they do. And it is found that every disturbing influence in the system is accounted for only by applying both parts of the law of gravity—that it *varies directly as the quantity of matter, and inversely as the square of the distance.*

130. *Paths of projectiles considered as orbits.*—When a stone is thrown, or a ball is fired, its path (undisturbed by the atmosphere) is part of an elliptic orbit, one of whose foci is at the center of the earth. In Mechanics, the path of a projectile is proved to be a parabola (Nat. Phil., Art. 44); but, in that demonstration, the vertical lines were assumed to be *parallel* to each other, and the force of gravity a *constant* force, neither of which is strictly true. Knowing the distance and period of the moon, the time in which a projectile would complete its revolution is found by Kepler's third law. Any force, which man could apply, would carry the lower extremity of the orbit so little beyond the center of the earth, that the mean distance might be called one-half the radius of the earth. Therefore, calling the moon's distance 60 radii, and its period $27\frac{1}{3}$ days, we have $(60)^3 : (\frac{1}{2})^3 :: (27\frac{1}{3})^2 : x^2$, from which x is found to be about 31 minutes. Every projectile, then, if it were free to complete its orbit unobstructed, and according to the law of gravity which prevails outside of the earth, would make an entire revolution, and return to its place, in about half an hour.

131. *Effect of increased velocity of projection.*—Suppose that P (Fig. 42) is a point near the earth, ADE, and that the velocity of projection, in the direction PB, is so greatly increased that the projectile strikes the earth at D. By a still greater increase of velocity it might meet the earth at E. In these cases the earth's center would be in the *most remote* focus of the orbit. But if we suppose the velocity so much increased that the centrifugal force just equals the force of gravity, then the body would describe the circular orbit PFG (Art. 90). As the mean distance now equals the radius of the earth, the time of revolution is found, by Kepler's third law, to be 1h. 24m. 39s. Any increase of the velocity of pro-

Fig. 42.



jection beyond this will again produce an ellipse, as PK whose *nearer* focus is at the earth's center. And we can imagine the velocity increased till the ellipse becomes one of extreme eccentricity, and then changes into the branch of a parabola, and then of a hyperbola, in which last cases the body will never commence a return toward the earth.

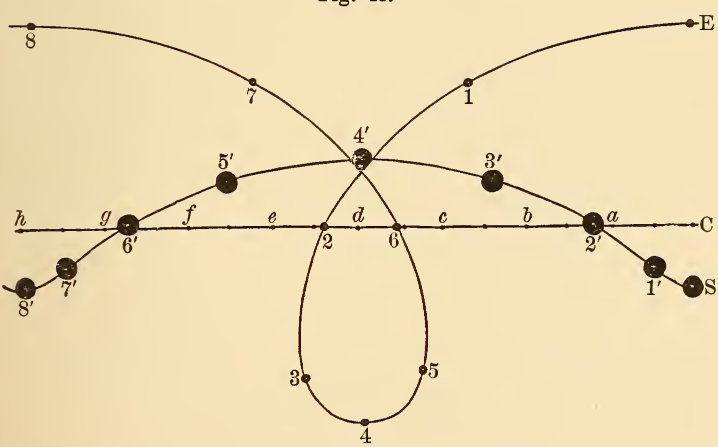
132. *Orbit motion and diurnal rotation by one impulse.*—If we suppose the projectile motion of the earth, or any other planet, to have been produced by a single impulse, that impulse may also have caused the diurnal rotation of the body. If the impulse had been directed in a line passing through the center of gravity of the planet, then it would have caused a progressive motion without rotation on an axis. But, if the line of impulse did not pass through the center of gravity, there would be rotation as well as progression. It has been calculated that the two existing rotations of the earth might have been produced by one impulse, applied in a line which passes 24 miles from the earth's center, on the side most remote from the sun.

Had it been directed through a point lying on the side nearest the sun, the diurnal motion would obviously have been retrograde.

133. *Motions of sun and planet, resulting from an impulse given to the planet.*—Suppose that the sun at S (Fig. 43), and the earth at E, mutually attract each other, and that an impulse is given to E in a line perpendicular to ES. S can not remain stationary and E revolve about it; for it is proved (Nat. Phil., Art. 89) that their center of gravity will move precisely as the sum of the bodies would move if united at the center, and the same impulse were applied to them. Suppose, for the sake of simplicity, that the weights of the bodies and the strength of the impulse are so related that the center, C, will pass over each unit of space, C*a*, *ab*, *bc*, etc., while E advances 45° in a circle about the moving center. Then, when the center is at *a*, E is at 1, 45° from a perpendicular at *a*. But S must be on the opposite side of *a*, and as far from it as from C before. Therefore, by the impulse given to E, and the mutual

attraction between E and S, the latter has been drawn along from S to 1′. Again, when the center is at *b*, E is at 2, and S at 2′. While E was on the upper side of C*h*, S was drawn toward that line, and now crosses it, and by its inertia continues upward, although E is now below the line. In this manner the bodies revolve about the moving center, describing *circles* relatively to that, but curves of a totally different character in space. These curves are always some variety or other of the class of curves called *epicycloids*. In the case represented in the figure, the planet describes an epicycloid which forms a series of loops, intersecting its own path at every revolution, while the path of the heavier body is of a waving form. The body E *retrogrades* on the lower part of the loop from 3 to 5, while S advances continually, but with unequal velocities, each body being alternately drawn forward and held back by the other.

Fig. 43.

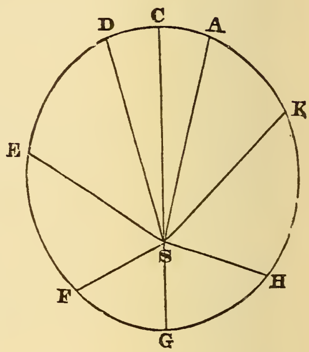


The only way in which two separate bodies could be made to rotate about a fixed center of gravity, would be to give an equal impulse to each body, and in opposite directions. Two such forces would constitute a couple (Nat. Phil., Art. 54), whose effect is to produce rotation merely.

134. *Why a planet at aphelion begins to return, or at perihelion begins to depart.*—It might be thought that a planet at

its aphelion, C (Fig. 44), being less attracted toward the sun than at any other point, would continue to withdraw, instead of commencing to return; and that when at its perihelion, G, being more attracted than elsewhere, it would continue to approach till it falls to the sun. The reason why a planet begins to return after reaching the aphelion is to be found in its diminished velocity. As the planet recedes through H, K, and A, the centripetal force toward S draws it back, and causes continual retardation, till at C the velocity is so much diminished that the attraction of S, though less than elsewhere, is still sufficient to curve the path so that it falls within a circle about the centre S, and the planet begins to approach the sun.

Fig. 44.



Again, as the planet passes through D, E, and F, the attraction toward S partly conspires with its inertia, and it is continually accelerated, till, at G, its velocity has become so great that its path strikes outside of a circle about the center, S, and it begins again to depart as before.

CHAPTER IX.

PRECESSION OF EQUINOXES.—NUTATION.—ABERRATION OF LIGHT.—APSIDES OF THE EARTH'S ORBIT.

135. *Precession of equinoxes described.*—The points in which the equator intersects the ecliptic on the celestial sphere are not stationary, but have a slow *retrograde* movement—that is, they revolve from east to west. The sun, therefore, in its annual progress eastward, crosses the equator each year a little further west than it did the year previous This motion is

called the *precession of the equinoxes*, either because the time of the equinoxes precedes the time in which the sun would have passed them if they had remained at rest, or because, in the daily transit of the meridian, the equinoxes precede those stars which crossed at the same time with them the preceding year.

The equinoctial points retrograde about $50\frac{1}{4}''$ each year. At this rate, it will require 25,800 years to make a complete circuit of the heavens.

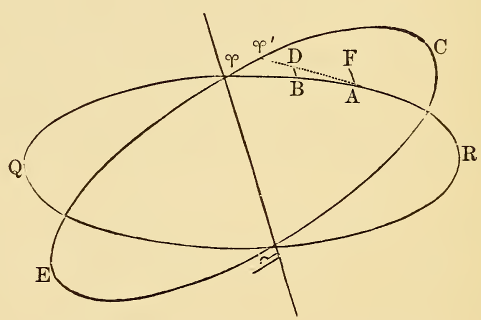
136. *Signs of the ecliptic displaced from the signs of the zodiac.*—The want of coincidence between the signs of the ecliptic and the signs of the zodiac was noticed (Art. 61). They coincided at the time the division was made, about 2,000 years ago; and the precession during this period has moved the equinoxes backward $2,000 \times 50\frac{1}{4}'' = 28°$, nearly. Hence, Aries of the zodiac almost coincides with Taurus of the ecliptic, Taurus of the zodiac with Gemini of the ecliptic, etc.

137. *Motion of the north and south poles.*—Considering the plane of the ecliptic as fixed, its poles of course occupy fixed positions among the stars. But this is not true of the poles of the equator. Their distance from the poles of the ecliptic is equal to the obliquity of the two circles—that is, 23° 27′. As this angle remains nearly constant, and the points of intersection move around westward, the poles of the equator must likewise move round those of the ecliptic in the same direction, and occupy the same period, 25,800 years in completing their revolution. The north pole of the equator is now near the star in Ursa Minor, known as the pole-star. According to the earliest catalogues, the pole was 12° distant from the pole-star. It is now somewhat more than 1° distant, and will, at the nearest, pass within $\frac{1}{2}°$ of it. In about 13,000 years the pole will be on the opposite side of the pole of the ecliptic, near the bright star α Lyræ, which will then be the pole-star.

138. *Cause of precession.*—The precession of the equinoxes is a disturbance produced by the sun's and moon's attraction upon the equatorial ring of the earth, as it rotates on its axis.

The sun being in the ecliptic, while the equatorial ring is inclined 23° 27′ to it, the sun's attraction is oblique to the plane of the ring; and one component of this force is perpendicular to the ecliptic. In most positions of the ring in relation to the sun, this component acts on one part to press it *towards* the ecliptic, and on another part to move it *from* the ecliptic. But the first is in excess; so that, on the whole, the ring tends to turn on the line of equinoxes towards the plane of the ecliptic. And this tendency, compounded with the inertia of the ring in its diurnal rotation, moves the equinoxes backward.

Fig. 45.



Let EC (Fig. 45) represent the plane of the ecliptic, and QR the equatorial ring of matter. A particle, A, of the ring, by its inertia of rotation, tends to move toward ♈ in the plane QR. Let AB represent this force, and AF the pressure toward EC, produced by the sun; then the resultant will be the diagonal AD, shifting the equinox back to ♈′. All the particles are subjected to this influence, except at the moment (each day) of crossing ♈ and ♎, so long as the sun itself is not in the line ♈♎ produced, which occurs in March and September. The effect is then interrupted for a time.

As the moon is always near the ecliptic—sometimes on one side of it, and sometimes on the other—its action on the whole conspires with that of the sun. And as it is comparatively near, though it is so small a body, its effect is more than twice as great as that of the sun. The planets produce a

very minute effect on the ring, tending to diminish the amount of precession. The joint effect of all the bodies mentioned is, as stated above, $50\frac{1}{4}''$.

139. *Law of composition of rotations.*—The case of precession of equinoxes is classed under the general law for the composition of two rotations, which is analogous to that for the composition of two rectilinear motions (Nat. Phil., Art. 38). It may be stated thus: *if two forces are applied to a body, which, separately, would cause rotation on two different axes, their joint action will produce rotation on a third axis lying in the plane of the other two, and making angles with them, whose sines are inversely as the forces.* In precession, the earth rotates on the diurnal axis by one force, and the sun and moon tend to rotate it on the line of the equinoxes. As the latter force is minute compared with the other, the new axis is shifted by a very small angle each year from the diurnal axis toward the line of equinoxes. And this line slides along the ecliptic, so that the two axes remain perpetually at right angles with each other.

The *rotascope*, a modification of Foucault's gyroscope, may be used to exhibit a very perfect illustration of the precession of equinoxes.

140. *Cause of the slowness of precession.*—If the equatorial ring were a separate body rotating about the earth in its own plane, its points of intersection with the ecliptic would retrograde very rapidly by the action of the sun and moon. The reason why the precession is exceedingly slow is, that while the disturbing action is exerted only on the ring, the force around the diurnal axis consists of the inertia of the entire earth. The ring can not move by itself, but must carry the whole mass of the earth with it.

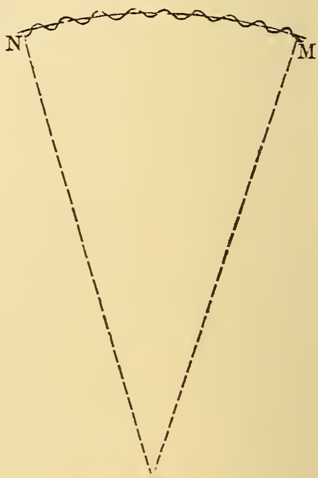
141. *The tropical and sidereal year.*—The fact of precession shows that the year has two different values, according as we reckon from a *star* or from an *equinox.* Hence, the *sidereal year* is defined to be the period occupied by the sun in

passing eastward around the heavens from a star to the same star again; and the *tropical year*, the time of passing around from an equinox to the same equinox again (Art. 86). As the equinox moves westward, the sun reaches it sooner than if it were stationary, and thus makes the tropical year shorter than the sidereal, by the time required to pass over $50\frac{1}{4}''$, which is 20m. 22.9s. As the tropical year is 365d. 5h. 48m. 46.15s. (Art. 86), the sidereal year, therefore, is 365d. 6h. 9m. 9s.

Though the sidereal year is the true period of the earth's revolution about the sun, yet the tropical year possesses by far the greatest interest, because it is the period in which the seasons are completed.

142. *Nutation.*—By precession alone, the pole of the equator would move in the circumference of a circle about the pole of the ecliptic. But this motion is modified by a minute vibration from side to side, as it advances, so that the line described by the pole is a delicate wave lying along on the circumference, as represented in Fig. 46, where P represents the pole of the ecliptic, and MN the path of the pole of the equator around it. This vibratory motion is called *nutation*. It is principally due to the *unequal* action of the moon upon the equatorial ring.

Fig. 46.



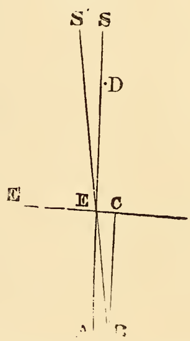
The moon's action, at any given time, tends to revolve the ring into the plane of its orbit. But, on account of the retrograde motion of its nodes, the angle between the ring and the moon's orbit varies from $18\frac{1}{2}°$ to $28\frac{1}{2}°$, going through all the changes every nineteen years. Owing to these changes of position, the equinoxes will recede sometimes faster, and sometimes slower; while the inclination of the equator to the ecliptic will also increase and decrease, causing the poles of the equator to oscillate, as stated

above. The amount, by which the pole of the equator moves to and from the pole of the ecliptic is 18″.

The waves in the figure are exceedingly exaggerated The arc MN being about $\frac{1}{10}$ of the circumference, the waves, if truly represented, would be small enough to cross the arc 270 times.

143. *Aberration of light.*—The heavenly bodies suffer a minute apparent displacement, on account of the progressive motion of light, combined with the earth's motion in its orbit. Suppose the earth to move from C to E (Fig. 47), while the light, coming from S, describes the line DE. If they arrive together at the point E, the impulse on the retina of the eye will not be in the same direction as if the observer had been at rest; but the light will appear to come in the direction S′E, the body being apparently thrown forward from S to S′. For, make EA = DE, and complete the parallelogram CA; and suppose, according to the principle of equal action and reaction, that the light has the motion EC given to it, in place of the earth's motion, CE; then the two motions, EA and EC, will produce the resultant, EB, as though the light had come from S′ instead of S.

Fig. 47.



144. *Aberration illustrated.*—The apparent direction of any kind of impulse is modified in the same way, by the motion of the person who receives it. For instance, if the wind drives drops of rain in a person's face, at a certain inclination, while he is standing still, when he comes to move toward the wind, they will strike him at a less inclination with the horizon, as though the source of the drops was further forward. For, when the person moves, the effect is the same as if he remained at rest, and the wind were to receive an increment of velocity equal to his motion.

145. *Greatest and least aberration.*—The greatest aberration occurs when the body, from which the light comes, is in a

direction at right angles to the line of the earth's motion The displacement is then 20″.5. When the earth is moving directly toward or directly from the body, the aberration is zero. Therefore, a star in the plane of the ecliptic is seen in its true place once every six months; but three months before and three months after either of those times, it is displaced 20″.5 in opposite directions, making the total arc of displacement 41″. But a star at the pole of the ecliptic, being always thrown forward of its true place by 20″.5, will seem to describe each year a circle, whose diameter is 41″. Between the ecliptic and its poles, the apparent orbit of aberration is an ellipse, whose major axis is 41″, and whose minor axis increases with the latitude of the body.

146. *Velocity of light computed by aberration.*—In the triangle AEB (Fig. 47), AB represents the velocity of the earth, AEB the observed aberration, and EAB the angle between the line of the earth's motion and the direction of light. When EAB=90°, the aberration is found to be 20″.4451. Therefore,

tan 20″.4451 : rad :: 18.393 miles : 185,600

miles per second, which is about the velocity of light.

147. *Advance of the apsides of the earth's orbit.*—It was intimated in Art. 74 that the line of apsides is not stationary. If the exact place of the perihelion among the stars be noted, it will be found the next year 11″.5 further east—that is, the apsides *advance* 11″.5 per year. But in longitude, the advance is much faster, since the vernal equinox, from which longitude is reckoned, retrogrades 50¼″ per year. The perihelion, therefore, increases its longitude nearly 62″ each year.

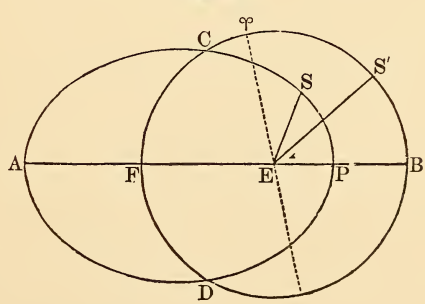
As the longitude of the perihelion in 1800 was 279° 30′ 8″ (that is, 9° 30′ 8″ past the winter solstice) it must have been just at the solstice in the year 1247. For, 9° 30′ 8″ ÷ 61¾″ = 553 years; and 1800 − 553 = 1247. In a similar manner, it is found that the perihelion will be at the summer solstice in the year 11741. In the course of many centuries, the length and temperature of the seasons are modified by these slow movements of the equinoxes and the apsides (Art. 75).

148. *Cause of the advance of apsides.*—The apsides of the earth's orbit are made to advance by the attraction of the heavy planets, whose orbits are outside of it. The entire resultant of the attractions of these planets upon the earth, is to diminish a little the earth's tendency to the sun. Hence, as the earth approaches one of its apsides, its path is not sufficiently drawn in by the sun to meet the former line of apsides at right angles. But it makes right angles with a radius vector a little further on, which becomes, therefore, the new line of apsides.

149. *Sun's anomaly.*—The sun's longitude is his distance eastward on the ecliptic from the vernal equinox (Art. 15). Its *anomaly* is its distance eastward, on the ecliptic, from perihelion. The reason for reckoning motion from the perihelion is, that the angular velocity depends on it; so that, to find the true longitude of the sun at any time, we need to know how far it is from the perihelion.

150. *How to find the true longitude of the sun at a given time.*—It is first supposed that the sun moves uniformly in a circle. And by knowing what its mean motion is, and how long it is since it passed the vernal equinox, we have its *mean* longitude at once. But this needs correction on account of

Fig. 48.



the variable motion in the ellipse. Let E (Fig. 48) be the earth; PCA, the elliptic orbit of the sun; and BCF, the sup-

posed circular orbit whose area equals that of PCA. Suppose the sun's mean place to be at S′, and the vernal equinox at ♈; then its mean longitude is ♈DS′, already obtained. The angle BES′ is its *mean anomaly.* But as the sun has been passing through the nearest part of its orbit, its true place is further advanced, as at S. The angle PES is the *true anomaly*, and the difference between them—that is, S′ES—is called *the equation of the center.* This equation, or correction, being found in tables of the sun's motions, and applied to the mean longitude, gives the true longitude.

If the mean and true places are considered as agreeing at P, then the equation of the center immediately becomes positive, and increases to its maximum at C; after which it diminishes, and the mean and true places agree again at A. After that, the sun falls behind its mean place, and the equation is negative, till the sun reaches P, the greatest value being at D.

The eccentricity of the earth's orbit is so small, that the sun's mean and true places never differ so much as 2°, the greatest equation of the center being 1° 55′ 27″.

151. *The anomalistic year.*—The perihelion is another point from which to measure the revolution about the sun. The time of passing round from perihelion to perihelion again is called the *anomalistic year.* It is 4m. 40s. longer than the sidereal year, or 365d. 6h. 13m. 49s.

CHAPTER X.

THE MOON.—ITS REVOLUTIONS.—ITS PHASES.—THE CONDITION OF ITS SURFACE.

152. *Distance and dimensions of the moon.*—The moon is a satellite of the earth, revolving about it within a comparatively small distance, and accompanying it in its orbit around the sun. The mean horizontal parallax of the moon at the

earth's equator being $57' \, 2''.7$, its mean distance is found by the proportion (Fig. 4),

$$\sin 57' \, 2''.7 : \text{rad} :: 3962.8 : 238{,}820\text{m}.$$

The moon's angular diameter is $31' \, 6''$; therefore, rad : sin $15' \, 33''$ :: 238,820 : 1080.3; which is the moon's semi-diameter in miles. Hence, the moon's diameter is 2,160.6 miles.

The *surfaces* of the earth and moon being as the squares of their radii, are as 13 : 1.

The *volumes* of the earth and moon being as the cubes of their radii, are as 49 : 1, nearly. But the moon's density is so small (3.4), that the *masses* are nearly as 81 : 1.

The force of *gravity* on the earth to that on the moon is as $\frac{81}{(3956)^2} : \frac{1}{(1080)^2} :: 6 : 1$, nearly.

153. *Revolution about the earth.*—The slightest attention to the position of the moon, from night to night, shows that it moves eastward, among the stars, several degrees every day. If the instruments of the observatory be employed to measure its right ascension and declination, as in the case of the sun (Arts. 58, 59), it is ascertained that the moon describes nearly a great circle, inclined about 5° to the ecliptic, and occupies 27.32 days in returning to the same place among the stars.

The inclination of the moon's orbit to the ecliptic varies from $5° \, 20' \, 6''$ to $4° \, 57' \, 22''$; but its mean value is $5° \, 8' \, 44''$.

154. *Months.*—The period just mentioned, in which the moon makes a revolution from a star to the same star again, is called the *sidereal month.* The time occupied in making a revolution relatively to the *sun*, instead of a star, is called a *synodical month.* This is more than two days longer than the sidereal month; for the moon's daily progress is about 13°; and during the 27 days of its revolution, the sun, at the rate of 1° per day, will advance 27°, requiring more than two additional days for the moon to overtake it.

The mean length of the synodical month is 29.53 days.

155. *Nodes.*—The points where the moon's path cuts the circle of the ecliptic are called the moon's nodes. The *ascending* node is the one through which the moon passes from the south to the north side of the ecliptic; the other, 180° from it, is called the *descending* node.

156. *The moon's positions in relation to the sun.*—The moon is said to be in *conjunction* with the sun, when both bodies have the same longitude; in *opposition*, when their longitudes differ by 180°. The conjunction and opposition are called by the common name of *syzygies.*

When the longitude of the moon is 90°, or 270° greater than that of the sun, it is said to be in *quadrature.*

The points midway between syzygies and quadratures are called *octants.*

The period in which the moon passes from any one of these points to the same point again—that is, a synodical month—is also called a *lunation.*

157. *To find the synodical month.*—The synodical month is best obtained by comparing ancient and modern eclipses. An eclipse of the sun takes place at the time of conjunction. If then, the whole interval between the recorded date of a solar eclipse, which occurred before the Christian era, and the time of another, which occurred recently, be divided by the number of intervening lunations, the quotient is a very accurate expression of the mean synodical month.

The mean synodical month, as thus obtained, is 29d. 12h. 44m. 3s. = 29.5306 days.

158. *To find the sidereal month.*—Dividing 360° by 365.25635, the number of days in a sidereal year, we have 0°.9856, the mean daily progress of the sun. Multiplying this by 29.53, the number of days in a synodical month, we find 29°.105, the arc passed over by the sun in that time. Now, the moon passes over 360° + 29°.105 in a synodical month, but only 360° in a sidereal month. Hence, we have the proportion, 360° + 29°.105 : 360° :: 29.53d. : 27.32d.

The sidereal month, more exactly, is 27d. 7h. 43m. 11s.

159. *Form of the moon's orbit.*—It is ascertained by the same method as was described (Art. 71), that the moon's orbit is an ellipse, one of whose foci is at the earth. The moon's apparent diameter varies from 33′ 31″ to 29′ 21″. Therefore, the greatest and least distances of the moon from the earth are in the ratio of these numbers, or as 8 : 7, nearly; and the eccentricity $= \frac{1}{15}$ or 0.067, which is about four times as great as the eccentricity of the earth's orbit (Art. 73). Yet a figure in the exact form of the moon's orbit could not be distinguished from a circle, since the major axis would exceed the minor by less than $\frac{3}{1000}$ of its length.

The point of the moon's orbit nearest the earth is called the *perigee*, the most distant point the *apogee.*

160. *The moon's diurnal motion.*—The moon not only revolves about the earth, but also on its own axis in the same length of time—that is, once in 27.32 days; and its axis is nearly perpendicular to the plane of its orbit. This rotation is indicated by the fact that the same side of the moon is always presented toward the earth. If it should pass around the earth, and not turn upon an axis, it would obviously present all sides to us in the course of each revolution.

But though it keeps the same side toward the earth, it presents all sides to the sun once in each synodical month; therefore, the days and nights on the moon are nearly 30 (29.53) times the length of those on the earth.

161. *The moon's librations.*—Though the same side of the moon is turned toward us on the whole, yet there are slight apparent oscillations, by which narrow portions of the other hemisphere alternately come into view. These are called *librations.* They are of three kinds: the libration in *longitude*, the libration in *latitude*, and the *diurnal* libration.

162. *The libration in longitude.*—By this libration we extend our view a little further round upon the moon's equator, first on one side, then on the other, every sidereal month.

It arises from the fact that while the moon rotates *uniformly* on its axis, it revolves in its elliptical orbit with *un-*

equal angular velocity. Near the apogee, where it moves slowest, it rotates more than 90° on its axis, while passing just 90° around us, and thus reveals a little of the remote hemisphere on the *eastern* side. Near the perigee, on the other hand, where the orbit motion is rapid, it makes less than one-fourth of a rotation, while going 90° around the earth. This brings into view a little of the other hemisphere on the *western* limb.

If the moon's orbit were a circle, there would be no libration of longitude.

163. *The libration in latitude.*—As the name implies, this libration extends our view alternately north and south on the moon's meridian. As the moon's equator is a little inclined to the plane of its orbit, its north and south poles are brought alternately toward us, just as the earth's poles are presented in turn toward the sun every year. The mean value of the inclination of the moon's equator to its orbit is 6° 39′.

If the moon's equator and its orbit were in the same plane, there would be no libration of latitude.

164. *The diurnal libration.*—This is the effect of diurnal parallax. When the moon is on the meridian, we view it nearly as from the center of the earth; but when it is at the horizon, we see it, as it were, from a position near 4,000 miles higher, and extend our vision a little distance over its western limb at rising, and its eastern at setting.

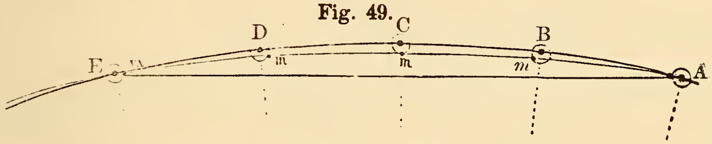
165. *Apparent diameter on the meridian and at the horizon.*—The distance of the moon from the earth is about 60 times the radius of the earth. Therefore, when the moon is on the meridian, as it is $\frac{1}{60}$ nearer than when at the horizon, its apparent diameter is $\frac{1}{60}$ greater. This change, equal to about 30″, is too small to be perceived by the eye, but can be measured by instruments.

166. *The moon's revolution about the sun.*—While the moon revolves about the earth, the earth revolves about the sun, at a distance 387 times as great. For, $238{,}820 \times 387 = 92{,}423{,}000$.

Therefore, the moon really has a third revolution—namely, that in company with the earth around the sun. And this is far greater than its other revolutions, which have been described. A point of the moon's equator, in its diurnal motion, goes only 10 miles per hour. Around the earth, the moon's velocity is nearly 2,300 miles per hour; but around the sun, it is more than 66,000 miles per hour.

167. *Form of path around the sun.*—Whenever a body revolves about a center, while that center is itself in motion, the body describes a species of curve, called an *epicycloid.* The moon's path about the sun is a *waving* epicycloid. Let the small circles at A, B, etc. (Fig. 49), represent the size of the

Fig. 49.



moon's orbit, and let AE be an arc of the earth's orbit, the sun being at the intersection of the dotted lines when produced. While the moon describes one half of its orbit, the earth goes over $\frac{1}{25}$ of its annual circuit—that is, from A to E. Therefore, the earth being at A, suppose the moon in quadrature on the left, beginning to describe the semicircle nearest the sun. When the earth reaches B, the moon has passed to the octant *m;* at C, the moon is in conjunction; at D, it is at the next octant; and at E, it is again in quadrature on the right, having described a semicircle relatively to the earth. But, in relation to the sun, it has passed over the curve inside of the earth's path, from A*mmm*E. At E, it crosses the earth's path, and while describing the outer semicircle, it advances with the earth a distance equal to AE, on the outside. Thus, the moon's path around the sun consists of 25 undulations, so slight that, if represented alone, the whole would scarcely be distinguished from the earth's orbit.

168. *By what forces the moon is mainly controlled.*—Since the moon describes around the sun an orbit at the mean dis-

tance of the earth's orbit, and in the same time, it must be subject to the same projectile and centripetal forces. If the earth, therefore, were to be annihilated, the moon's path about the sun would not be essentially disturbed; the waves only would cease, and the orbit become an exact ellipse.

The relative attractions of the earth and sun, exerted on the moon, are estimated by the formula proved in Art. 92, $c \propto \frac{r}{t^2}$. Considering the radius of the moon's orbit = 1, that of the earth's orbit is about 387; and the times are 27.32d. and 365.25d., respectively. Hence, attraction to the earth : that to the sun : : $\frac{1}{(27.32)^2} : \frac{387}{(365.25)^2}$: : 1 : 2.2, nearly. Therefore, the sun, though so very far from the moon, exerts upon it $2\frac{1}{5}$ times more attraction than the earth does.

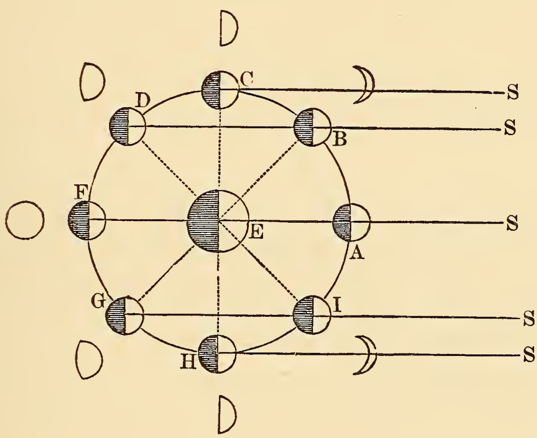
169. *How the earth's action causes the waves in the moon's path.*—When the moon is in conjunction, as at C, the earth draws it away from the sun, so that it begins to move further off, as at D, E, etc., till it reaches opposition. But, at opposition, the earth is on the same side as the sun, and increases the moon's tendency toward it, so that the moon begins to move toward the sun, and continues approaching till it reaches conjunction again. But, in describing the wave line, the moon sometimes gets in advance of the earth in its orbit, as at A, and then falls behind, as at E. For, the earth at A draws the moon *backward*, and it falls further and further back, till it is behind the earth in its motion, as at E, where the earth, having overcome the backward motion, draws it forward, till it passes by, and is again in advance of the earth. Thus, in the moon's great revolution around the sun, we may regard its path as thrown into the waving line by the small disturbing influences of the earth.

170. *Phases of the moon.*—The moon is not self-luminous, and is seen only as it reflects to us the light which falls upon it. The several forms which the part illuminated by the sun presents to our view, are called *phases.*

The *circle of illumination*, or the *terminator*, is the circle

which separates the hemisphere enlightened by the sun from the dark hemisphere, and is perpendicular to the sun's rays which fall on the moon. The *circle of the disk* is that which separates the hemisphere turned toward the earth from the opposite one, and is perpendicular to our line of vision. The phase depends on the size of the angle formed at the moon, between the solar ray and our visual line.

Fig. 50.



Let the earth be at E (Fig. 50), and the moon in several positions, A, B, etc., and let the lines AS, BS, etc., be directed toward the sun. At A, the moon is in conjunction, and wholly invisible—this is called *new moon;* and the angle SAE, between the solar ray and visual ray, is 180°. From A to C (as at B), the phase is called *crescent;* and the angle, SBE, is obtuse. The *first quarter* occurs at C, the quadrature, where SCE is a right angle. From C to F (as at D), the phase is called *gibbous;* in this phase, the angle, SDE, is always acute. At F, the moon is in opposition, and wholly illuminated. This is called *full moon;* the angle, SFE, is 0°. From F to A, the phases are repeated in reverse order, the *last quarter* being at H. The outer figures at B, C, etc., show the corresponding phase

171. *The meridian altitudes of the moon at a given phase.*—It is generally observed that at a given age of the moon, for instance at the full, its meridian altitude is very different at different seasons of the year. This is readily explained, by noticing the moon's relations to the sun. As the moon's path is everywhere near the ecliptic, the *new moon* will culminate at a high point when the sun does—that is, in the summer. But, in the same season, the *full moon*, being opposite to the sun, will culminate low. On the contrary, when the sun is in the most southern part of the ecliptic, and culminates low, as is the case in winter, the new moon will do so likewise; but the full moon will culminate at a high point. In the polar winter, therefore, when the sun is absent for months, the moon, whenever near the full, circulates round the sky without setting.

172. *The harvest moon.*—This name is given to the full moon which occurs nearest to the autumnal equinox, September 22d, and which rises from evening to evening with a less interval of time than the full moon of any other season.

The sun being at the autumnal equinox, the moon is near the vernal equinox, and at sunset, the southern half of the ecliptic is above the horizon, and makes the smallest possible angle with it. It is this small angle, made by the ecliptic, and therefore by the moon's orbit with the horizon, which causes the small interval in the time of the moon's rising from one evening to another; for, as the moon advances 13° each day in its orbit, this arc is so oblique to the horizon that its two extremities rise with only a few minutes' difference of time; but the *place* of rising moves rapidly northward.

The harvest moon attracts most attention in high latitudes, where the angle between the ecliptic and horizon is smaller, and therefore the intervals of time are less.

The moon passes the vernal equinox every month, and therefore rises with the same small intervals. But when the moon is not *full* at the same time, the circumstance is unnoticed.

173. *Inequalities of the moon's surface.*—These are clearly

revealed by the changing direction of the sun's rays. As the terminator advances over the disk, the light strikes the highest peaks, which appear as bright points a little way upon the dark part of the moon. After the terminator has passed over them, they project shadows away from the sun, which correspond to the apparent shape of the elevations, and grow shorter as the rays fall more nearly vertical. And again, in the waning of the moon, the shadows are cast in the opposite direction, lengthening until the dark part of the disk reaches them, and the summits once more become isolated bright points, and then disappear. Fig. 2, Frontispiece, will give some idea of these appearances.

174. *Forms of valleys.*—The most striking characteristic of the moon's surface is its numerous circular valleys. A few are represented in Figs. 1 and 2, Fr. The smaller and more regular ones are of all sizes, from one or two miles in diameter up to sixty miles. These are numbered by hundreds. The mountain ridge which surrounds one of these cavities is a ring, very steep and precipitous on the inner side; but externally it falls off by a rugged but gradual slope. These ridges are called *ring-mountains*. In the central part of the cavity are generally one or more steep, conical mountains. Some of the principal ring-mountains are No. 1. Tycho; 2. Kepler; 3. Copernicus; etc. (Fig. 1, Fr.)

There is another class of larger but less regular cavities, sometimes called *bulwark plains*. Their diameters are often more than one hundred miles. These are also surrounded by rough mountain masses arranged in a circle. Over these plains are sparsely scattered small conical and ring mountains.

There are still larger tracts, more level than the general lunar surface, and of a darkish hue, which still retain the name of *seas*, formerly given them, though they are covered with permanent inequalities, and show no signs of being fluid. Examples of these are: A, *mare humorum*; B, *mare nubium*, etc. (Fig. 1, Fr.)

Besides the ridges of mountains inclosing the circular val-

leys, there are occasional chains and spurs, having more resemblance to terrestrial ranges.*

175. *Luminous radiations.*—At full moon, all shadows disappear, because the light falls in the direction of our line of vision. But at that time another peculiarity presents itself. From a few of the large ring-mountains there radiate a great number of luminous stripes, nearly in straight lines, and extending, in some cases, hundreds of miles. They are not ridges, as they cast no shadows when the terminator passes them; and the difference of illumination must result from the different nature of their material. They are sometimes called *lava-lines.* The most extensive system occurs around Tycho, marked 1, in Fig. 1, Fr.

176. *Surface rigid and angular.*—Every part of the moon's surface has the appearance of rocky hardness. The interior slopes of the ring-mountains are steep, rough, and angular. The conical peaks within them appear like isolated rocks, resembling the needles of the Alps. The surface nowhere gives indication of having been softened down by the action of water.

177. *Probable volcanic origin.*—The circular cavities, with steep and rugged sides, appear like vast craters, and the mountains within them like volcanic cones, more recently thrown up. Nearly every part of the hemisphere presented to our view exhibits these indications of former volcanic action, on a scale far beyond any thing on the earth. But there is no evidence of volcanic action at present.

178. *Height of lunar mountains.*—One method of measuring the height of a lunar mountain is the following. Let the light from the sun, S (Fig. 51), pass the moon's surface at O, and illuminate the summit of the mountain, MF. To the observer on the earth at E, M is seen as a bright point beyond the ter-

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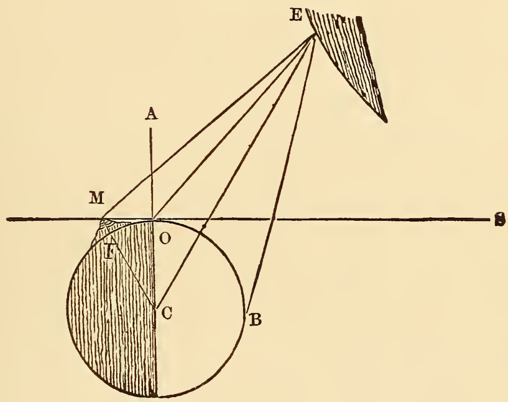
* The lunar map of Beer and Madler, $2\frac{1}{2}$ feet in diameter, contains a very perfect delineation of the mountains and valleys of the moon, accompanied by their names.

minator O. Let OEM, subtended by OM, be measured with a micrometer; also, OEB, between the terminator and the opposite edge of the disk. From the latter subtract CEB, the semi-diameter, and OEC is known; which, with OC and EC, will give EO and EOC. The supplement EOA, plus 90°, equals EOM. Then EO and the angles E and O, will furnish OM; from which and OC, CM is computed. FC, subtracted from CM, leaves FM, the height required.

Fig. 51.



The height of a mountain may also be determined by measuring the length of its shadow, and the inclination of the solar ray which casts it.

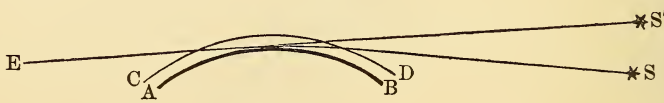
The highest of the lunar mountains have an elevation of $4\frac{1}{2}$ miles, and great numbers of them exceed three miles. Thus, the mountains of the moon are proportionally much greater than those of the earth. For, while the diameter of the moon is not much more than one-fourth as great as the earth's diameter, its mountains are about equal in height to the mountains on the earth.

179. *No atmosphere or vapor.*—If any kind of atmosphere were spread over the disk of the moon, it would reflect the sun's light so strongly as to dim the features of the solid surface. Nothing of the kind is ever perceived. No terrestrial

objects, however near, ever exhibit greater sharpness of outline than the inequalities of the moon; and they never vary in this respect, except in a manner which is obviously occasioned by our own atmosphere.

But the severest test of a perceptible atmosphere would be the effect on a star at the beginning and end of its occultation by the moon. Let AB (Fig. 52) be the edge of the moon's

Fig. 52.



disk, and CD that of the atmosphere around it. The light from the star S will, according to the laws of optics, be refracted toward the moon in entering its atmosphere, and as much more in the same direction in leaving it; so that it will reach the observer at E, appearing to come from S′, when the star is really behind the moon at S. Thus, it will appear to be detained in its diurnal motion as it approaches the edge of the moon, and to arrive only to S′ when it has really reached the position S. So, also, in reappearing at the opposite limb, the star will seem to have advanced to the edge, when it is still behind the moon; so that, after coming into view, and before passing by the atmosphere, it will again appear to be detained in its diurnal motion. Since it disappears too late, and reappears too early, the duration of occultation is too short.

Besides this irregularity in its motion, its brightness will also be a little dimmed by the obstruction of the atmosphere, just before disappearing, and just after reappearing.

Now, the nicest observations have failed to show either of these effects. The diurnal motion is uniform up to the very edge of the disk, and the actual continuance of occultation is equal to the calculated duration. And, as to loss of light, the star at its full brightness disappears all at once, with a suddenness which is startling. Its reappearance is equally sudden, and without any change of intensity in its light. The moon, therefore, has no appreciable atmosphere.

180. *Changes of temperature on the moon.*—The moon's equator makes an angle of only $1\frac{1}{2}°$ with the ecliptic, and therefore experiences no perceptible change of seasons; but its diurnal rotation is so slow, that the extremes of heat and cold during each day are excessive. A place on the moon is exposed to the full power of the sun's rays for about two weeks, and then is for as long a time turned away from the sun, without clouds, or even air, to prevent the free radiation of heat.

181. *View of the earth from the moon.*—

1. *As to magnitude.*—The apparent dimensions of the two bodies, as seen one from the other, are proportional to their real dimensions. Hence, in diameter, the earth as seen from the moon is $3\frac{3}{4}$ times as large as the moon viewed from the earth, and in area is about 13 times as large.

2. *As to phase.*—It is obvious, from Fig. 50, that when the full moon is presented to the earth, the earth's dark side is toward the moon, and the reverse. Also, that when we see the gibbous phases of the moon, a spectator on the moon would see crescent phases of the earth; for the angle SED or SEG would then be obtuse. In like manner, the relative phases are in every case supplementary to each other. This relation explains the well-known fact that near the time of new moon, the part of the moon not directly enlightened by the sun is distinctly visible. It is then illuminated indirectly by the earth, which is nearly full as seen from the moon, and reflects a strong light upon it.

For the same reason, the moon can be faintly seen in a total solar eclipse.

3. *As to position in the sky.*—The earth seen from the moon has no apparent diurnal rotation, as all other heavenly bodies have, but remains nearly fixed in the same part of the sky. This is owing to the fact that the moon's monthly motion and its diurnal motion are at the same rate in the same direction, so that one apparent motion of the earth neutralizes the other. Hence, a spectator occupying the middle of the moon's disk sees the earth perpetually near his zenith. Another, at the edge of the disk, sees it always near the same point of the horizon.

The first and second librations of the moon, since they vary the spectator's position a little in relation to the disk, merely cause small oscillations of the earth's place in the sky.

4. *As to surface.*—The earth, by its rotation, presents all its parts to the view of the nearer hemisphere of the moon once in 25 hours. To the other hemisphere it never appears at all.

On account of its nearness, and its great size, we might suppose that the geographical features of the earth would be very conspicuous to a spectator on the moon, and that the nature of its surface in nearly all respects could be thoroughly observed. But the deep and dense atmosphere of the earth would reflect an intense light, so as probably to render the inequalities of the terrestrial surface nearly invisible; and whenever clouds prevail over a country, that portion of the earth's surface would, of course, be entirely hidden from view.

CHAPTER XI.

DISTURBANCES OF THE MOON'S MOTION CAUSED BY THE SUN.

182. *Why the sun disturbs the moon's revolutions around the earth.*—If the sun were at an infinite distance from the earth and moon, however great its attraction might be, it would not disturb their mutual relations, because it would act on both exactly alike. Though the sun's distance from them is very great, being 387 times their distance from each other, yet the difference of action is sufficient to produce sensible disturbances. These disturbances are caused in part by difference of *distance*, and in part by difference of *direction*.

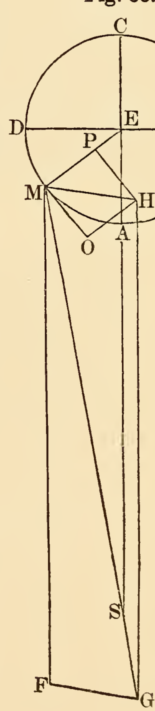
183. *The moon's gravity diminished at syzygies, and increased at quadratures.*—When the moon is in conjunction, the sun attracts it more than it does the earth, in the ratio of $387^2 : 386^2$, and thus diminishes the moon's tendency to the earth. In opposition, the sun attracts the moon less than it does the earth, in nearly the same ratio, which, as before, di-

minishes the moon's tendency to the earth. Therefore, at the syzygies, the moon's gravity to the earth is diminished. And the diminution is computed to be about $\frac{1}{90}$ of the whole.

In quadrature, the sun attracts the moon in a line slightly oblique to that in which it attracts the earth. Hence, there is a small component of its action directed *toward* the earth. Therefore, at the quadratures, the moon's gravity to the earth is increased. This increase is proved to be about $\frac{1}{180}$ of the whole, or one-half as great as the diminution at syzygies.

As the diminution at syzygies is more than the increase at quadratures, the entire effect of the sun's influence is to diminish the moon's gravity to the earth, and thus cause it to revolve in a larger orbit than it would do if the sun did not exist. The moon's gravity to the earth is diminished by $\frac{1}{360}$, in consequence of the sun's action.

Fig. 53.



184. *The sun's disturbing effect represented geometrically.*—Let ABCD (Fig. 53) be the moon's orbit described about the earth E, and S the place of the sun. Suppose the moon at M. Let ES represent the attraction of the sun upon the earth. Then (Art. 128), $SM^2 : SE^2 :: SE : \frac{SE^3}{SM^2}$ = the attraction of the sun upon M, in the direction MS. Make $MG = \frac{SE^3}{SM^2}$, draw MF equal and parallel to ES, and complete the parallelogram MFGH. Resolve the force MG into MF and MH. Since the component MF is equal and parallel to ES, which is the sun's attraction on the earth, it produces no disturbance; and the only force which can disturb the relations of M and E is the other component MH. This line lies in various positions, and is of various lengths, according to the place of M. It is convenient to reduce it to two ele-

ments called the *radial* and the *tangential* disturbing forces Draw MO tangent to the orbit, and EM joining the earth and moon; then, MH may be resolved into the radial force MP, increasing or diminishing the moon's gravity to the earth, and the tangential force MO, which increases or diminishes the velocity of the moon. In the figure, the position of MH is such, that MP increases the gravity, and MO accelerates.

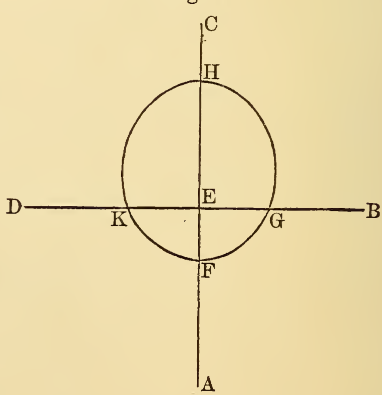
Near the quadratures, MP acts toward E; and near the syzygies, it acts away from E. MO accelerates on the quadrants DA and BC, and retards on AB and CD.

185. *Equations for correcting the moon's place.*—The moon's path being elliptical, and its motion being subject to several disturbances, its true longitude for a given time can not be found, except by applying various corrections.

186. *The equation of the center.*—First suppose the moon to revolve uniformly in a circular orbit, and then, as in the case of the sun (Art. 150), apply the equation of the center to change its place for the variable motion in the ellipse. The moon's orbit being more eccentric than the earth's, its greatest equation of the center is 6° 18′ 17″, while the sun's is less than 2°.

187. *Evection.*—A correction must be applied on account of the change of eccentricity caused by the sun's disturbance. This change of eccentricity is called *evection.* It is caused by the radial disturbance MP (Fig. 53), which produces greater or less effect, according to the position of the line of apsides in relation to the line of syzygies. Let FH (Fig. 54) be the line of apsides of the moon's orbit about the earth,

Fig. 54.



E, and suppose the sun to be in the direction A. Then AC is the line of syzygies, and the two lines coincide. The moon's gravity toward E is diminished at F and H, as it always is when in the line of syzygies. But at F, it is diminished less than ever, because there is the least difference of distances, AE and AF; while at H, it is diminished more than ever, because the difference of distances AE and AH is the greatest possible. Hence, F is *less* separated from E, and H *more* separated from E, than in any other situation. The same would be true, if the sun were in the direction of C. Therefore, when the line of apsides coincides with the line of syzygies, the moon's orbit is most eccentric.

Again, suppose the sun to be in the direction B or D; in other words, that the line of apsides is in quadrature. Then, the gravity of the moon toward E is increased at F and H, as it always is when in quadrature. But at F, its increase is the least possible, because the obliquity of FB to EB is the least possible; while at H, the increase is the greatest, because the obliquity of HB to EB is the greatest. Hence, HE is less, compared with FE, than in any other position. Therefore, the eccentricity is least when the line of apsides is in quadrature. The greatest correction for evection is 1° 12′.

188. *Variation.*—Another correction is applied on account of the alternate changes of velocity caused by the sun. This change of velocity is called *variation.* It is produced by the tangential disturbance MO (Fig. 53). From D to A, it conspires with the motion of the moon, and accelerates it. From A to B, it is directed backward, and retards the moon's motion. From B to C it accelerates, and from C to D it retards. It might be supposed that because the sun attracts toward S, this would act *against* the moon's motion in going from B to C, and thus retard it; and *with* it from C to D, and accelerate it. But the disturbing action is not the absolute, but the relative attraction. From B to C, the sun attracts the moon less than it does the earth; and the effect is the same as if it exerted no attraction on the earth, and urged the moon in the opposite direction—that is, toward C. Hence, the moon's velocity is alternately accelerated and retarded in the successive quad

ants, causing the greatest equation about 45° from the quadratures B and D. The variation at its maximum is about 37′.

189. *Annual equation.*—This is a change in the moon's motion, arising from the greater and less distance of the sun at different seasons of the year. The disturbing action of the sun is greatest when it is nearest—that is, at perihelion; and it is least when it is most distant, or at aphelion. This inequality is called the *annual equation*, since it passes through all its changes in a year. It amounts to about 11′.

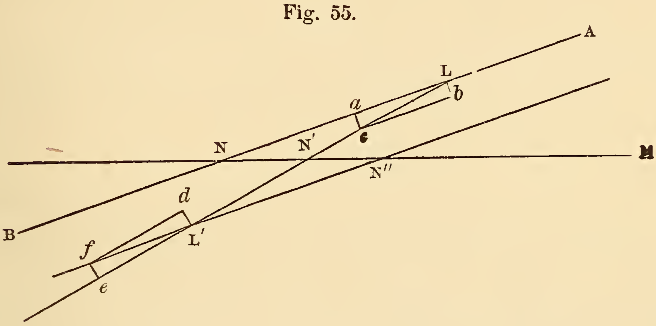
190. *Smaller equations.*—The foregoing are the largest inequalities of the moon's motion, which require corrections to be made for finding its true place. There is a large number of smaller ones, for which allowance must be made, in order to obtain the moon's longitude for a given time, with the utmost exactness. By the most complete tables of the moon now in use, its place can be determined within 3″.

191. *Apsides of the moon's orbit.*—The line of apsides *advances*—that is, moves forward—from west to east. This is a disturbance produced by the sun, and is explained in the same manner as the advance of the earth's apsides (Art. 148). The attraction of a body external to the orbit always tends to produce this effect. Though the sun makes the moon's gravity to the earth sometimes greater, and sometimes less, yet it, on the whole, diminishes it (Art. 183). Without any disturbing influence, the moon would always describe the same elliptic orbit. But as it approaches one of its apsides, it is, in general, not sufficiently drawn in toward the center to cut the former line of apsides at right angles; but it makes right angles with a radius vector a little further on, which, therefore, becomes the new line of apsides. The apsides of the earth's orbit advance with exceeding slowness (Art. 147); but the sun's disturbing power is so great, that those of the moon's orbit shift their place more than 3° in each sidereal month, and, therefore make a complete revolution in about 9 years.

192. *Retrogradation of the moon's nodes.*—In the pre-

ceding articles we have considered the disturbing force of the sun upon the moon *in* the plane of its orbit, one component being the *radial*, and the other the *tangential*. As the moon's orbit does not coincide with the ecliptic, the sun exerts another disturbing force—namely, *out of* the moon's orbit either to or from the ecliptic. This third force is called the *orthogonal* component. It causes a motion of the nodes, and a change of inclination.

Fig. 55.



Let MN (Fig. 55) represent a short arc of the ecliptic, AB an arc of the moon's orbit, and ANM the inclination of their planes. When the moon is at L, moving toward the node, N, the sun attracts it in a line slightly oblique to its orbit. Therefore, while one component of this disturbing force lies in the plane of the orbit, the other is perpendicular to it. Let L*b* be the distance through which the latter would move the moon in the time of its describing L*a* in its orbit. The resultant is L*c*, cutting the ecliptic in N'. Again, after passing the node, let the orthogonal component move the moon over L'*d*, while it would describe L'*e* in its own plane. Then, by the joint action of L'*e* and L'*d*, it describes L'*f*, which produced makes the node at N''. In the case here described, the line of nodes is supposed to lie perpendicular to the line joining the earth and sun, and we see that the node is made to move *backward*, both when the moon approaches it, and when it is leaving it. But in other positions of the line of nodes, it can be shown that the orthogonal component is directed sometimes *toward* the ecliptic and sometimes *from* it. In the former case the nodes retrograde, in the

latter they advance. In any revolution, however, the latter effect is less than the former; so that on the whole, the nodes have a retrograde motion.

The nodes of the moon's orbit retrograde at the rate of 19° 35′ each year, thus completing a revolution in 18.6 years.

193. *Disturbance of the inclination of the moon's orbit.*—When the moon approaches a node, the inclination of its orbit to the ecliptic is generally increased; for LN′M is greater than the interior angle LNM. And it is generally diminished as the moon leaves a node, since lN″N is less than the exterior angle lN′N. These alternate changes nearly balance each other, and leave the mean value of the inclination almost constant—namely, 5° 8′ 44″ (Art. 153).

194. *Periodical and secular inequalities.*—The inequalities in the moon's motion, which have been described, pass through all their changes in a short period, as a month, a year, or a few years at most. These are called *periodical.* But there are others, whose periods extend through many centuries or ages. These are called *secular.* Some minute secular disturbances in the solar system run on in the same direction for an indefinite number of centuries.

195. *The acceleration of the moon's mean motion.*—This is an interesting example of secular inequality. The period of a lunation is now sensibly shorter than it was before the Christian era. This is ascertained by comparing the recorded date of an eclipse which occurred in 720 before Christ with the time of any recent eclipse. The whole interval, if divided by the *present* mean length of a lunation, leaves a considerable remainder. The acceleration amounts to about 10″ in a century.

196. *Its cause.*—It has been stated that the sun diminishes the moon's gravity toward the earth (Art. 183). The amount of this diminution depends, in part, on the eccentricity of the earth's orbit. From the time of the earliest observations, the earth's orbit has been slowly approaching a circle, and will

continue to do so for many centuries to come. So long as the eccentricity of the earth's orbit is diminishing, the sun's disturbing action on the moon diminishes also. The moon, therefore, being less drawn away from the earth, describes a smaller orbit, and, consequently, in a shorter time. In the course ot ages, the earth's orbit will reach the limit of its change, and begin to grow more eccentric. The moon's orbit will then commence to enlarge, and will, therefore, require a longer time to be described.

CHAPTER XII.

ECLIPSES OF THE MOON.—ECLIPSES OF THE SUN.

197. *General relations in eclipses.*—The *moon* is eclipsed, when it is obscured wholly or in part by the earth's shadow. It can occur, therefore, only at opposition, or full moon. The *sun* is eclipsed, when it is either wholly or partially concealed from view by the moon coming between it and the earth. This can happen only at conjunction, or new moon.

If the moon's orbit and the ecliptic were coincident planes, there must be an eclipse of the moon at every full moon, and an eclipse of the sun at every new moon; for at those times the three bodies would be in a straight line. But as the moon's orbit and the ecliptic make an angle of 5° with each other, the moon generally passes opposition and conjunction so far north or south of the sun, that there is no eclipse. That an eclipse may occur, the syzygies must happen near the line of nodes, so that, as the moon comes into conjunction or opposition, some parts of the three bodies may be in a straight line.

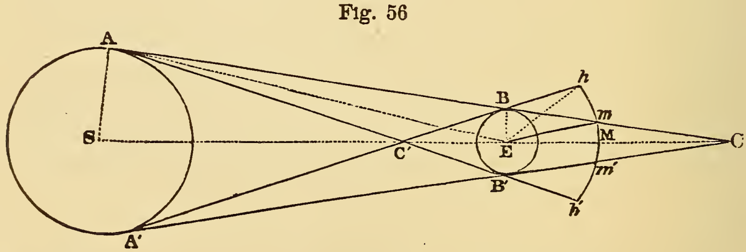
198. *Eclipse months.*—As there are two nodes on opposite sides of the heavens, the sun in its annual progress must pass through both of them every year, at intervals of about six months. And as the moon comes into the line of syzygies every two weeks, the sun will certainly be near enough to a

node for one or two eclipses, and possibly for three, every six months. Thus, the eclipses of any year always occur in *clusters*, at opposite seasons. If two or three are in January, the others are in July. These are called the *node months* of that year. In 1884, for example, the node months are parts of March and April, and parts of September and October. On account of the retrograde motion of the nodes, the sun passes from a node to the same one again in less than a year, so that the node months occur earlier each successive year perpetually.

199. *Eclipse of the moon.*—When the moon is eclipsed, there is nothing interposed to hide it from our view; but it merely falls into the shadow of the earth, and is obscured. This obscuration may possibly continue for several hours.

200. *Form and angle of the earth's shadow.*—As the sun is larger than the earth, and both are spheres, the tangents drawn from one to the other, along the corresponding edges, will converge and form a cone. Thus (Fig. 56), let AA′ be the sun, and BB′ the earth; then BB′C is the conical shadow; and rays of light from AA′, moving in straight lines, can not enter any part of it. The *axis* of the shadow, EC, is the extension of the line joining the centers of the sun and earth. Since the light is entirely excluded from the cone BB′C, it is often called the *total shadow.*

Fig. 56



Join AE; then the exterior angle AES = ACE + EAC; ∴ ACE = AES − EAC. But AES is the sun's apparent semi-diameter, and EAC is the sun's horizontal parallax.

Therefore, *the semi-angle of the earth's shadow is equal to the sun's apparent semi-diameter diminished by its horizontal parallax.* Calling the sun's semi-diameter δ, and its horizontal parallax p, the semi-angle of the shadow is $\delta - p$. $\delta = 16' \ 2''$, and $p = 8''.8$; $\therefore \ \delta - p = 15' \ 53''.2$, the mean value of the semi-angle of the shadow.

201. *Length of the earth's shadow.*—In the triangle ECB right-angled at B, as we know EB and ECB, EC is found by the proportion, sin $(\delta - p)$: rad :: 3956 : 856,050, the length of the earth's shadow in miles.

Since the moon is 238,820 miles from the earth, the length of the earth's shadow is more than $3\frac{1}{2}$ times the distance from the earth to the moon; and the moon, when eclipsed, passes through the broader part of it.

202. *Angular breadth of the section traversed by the moon.*—Let $h'h$ be a part of the moon's orbit supposed to pass through the axis of the shadow at M. Then Mm is the semi-diameter of the section, and MEm its angular semi-diameter, which is to be found. The exterior angle EmB = ECm + CEm; $\therefore$ CEm = EmB − ECm. But EmB is the horizontal parallax of the moon, and ECm the semi-angle of the shadow. Call the moon's parallax P, then the angular semi-diameter of the shadow $= \text{P} - (\delta - p)$, or $\text{P} + p - \delta$.

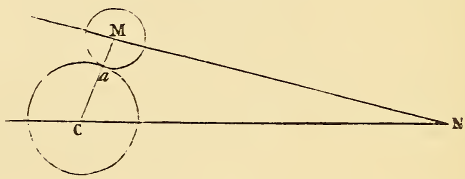
$\text{P} = 57' \ 3''$, and $\delta - p = 15' \ 53''$; $\therefore \ \text{P} + p - \delta = 41' \ 10'$, the mean semi-diameter of the section.

Since the moon's semi-diameter is $15' \ 33''$, the breadth of shadow where the moon crosses it is $2\frac{2}{3}$ times the breadth of the moon.

203. *Lunar ecliptic limit.*—The distance of the center of the earth's shadow from the node, when the moon at opposition would only touch the shadow, is called the lunar ecliptic limit. Let CN (Fig. 57) be an arc of the ecliptic, MN an arc of the moon's orbit, N the node, Ca the semi-diameter of the shadow, and aM that of the moon when it only touches the shadow at opposition. Then CN is the ecliptic limit. In the spherical triangle CMN, right-angled at M, N being known,

and also Ca and aM, we have, by Napier's rule, rad $\times$ sin CM = sin CN $\times$ sin N; from which CN is obtained. Since N, Ca, and aM are all variable, CN must also vary. Its greatest value is 12° 24′, beyond which an eclipse is impossible. Its least value is 9° 24′ within which an eclipse can not fail to occur.

Fig. 57.



204. *Magnitude of eclipse.*—The mere contact of the moon and earth's shadow at the ecliptic limit is called an *appulse*. If the moon is obscured only in part, the phenomenon is called a *partial eclipse*. It is a *total eclipse* when the moon is entirely enveloped in the shadow. If its center passes through the axis of the shadow, there is a *central eclipse*.

205. *The earth's penumbra.*—If tangents be drawn across the opposite sides of the sun and earth, as Ah', A′h (Fig. 56), they diverge, and inclose a space around the total shadow, called the *penumbra*, or partial shadow. Its form is the frustum of a cone, and it extends to an infinite distance beyond the earth. Within the penumbra, and outside of the shadow, there is light from a part of the sun only, while the other part is concealed by the earth. Thus, at a point between BC and Bh produced, it is obvious that the limb of the sun near A could not shine, because the light would be intercepted by the opposite side of the earth near B. The vertex of the penumbra is between the earth and sun, at C′.

206. *Dimensions of the penumbra.*—The semi-angle of the penumbra is h'C′C (Fig. 56), which is equal to AC′S. And the external angle AC′S = EAC′ + C′EA. But EAC′ is the sun's horizontal parallax = p; and C′EA is the sun's apparent semi-

diameter = δ; therefore, the semi-angle of the penumbra = $p + \delta$.

The semi-diameter of the section of the penumbra through which the moon passes is hM, and its angular or apparent semi-diameter is hEM. And hEM, being external to the triangle hEC′, equals EC′h + EhC′. But EhC′ is the moon's horizontal parallax = P; and EC′h = $p + \delta$; therefore, the apparent semi-diameter of the earth's penumbra = $P + p + \delta$ At mean values, this equals 1° 13′ 14″, which is nearly 5 times the semi-diameter of the moon.

207. *Effect of the penumbra.*—On account of the penumbra, the edge of the total shadow is not sharply defined, but shades off into the full light by slow degrees, so that the moon passes over rather more than its own breadth after entering the penumbra, before it reaches the total shadow. This circumstance renders the exact moment of the observed beginning or end of a lunar eclipse uncertain.

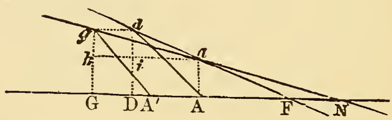
208. *Effects of the earth's atmosphere.*—It is found, by calculation, that the sun's light which traverses the lowest parts of the earth's atmosphere would be so much refracted as to meet the axis of the shadow before reaching the moon. Hence, the whole disk of the moon is *visible*, even in a central eclipse, and appears of a dull red color.

Another effect is the enlargement of the shadow. The light, which passes the earth near its surface, and would immediately surround the shadow if there were no atmosphere, is, in part, obstructed, and in part diffused through the whole breadth of the shadow, as just stated. Therefore, the boundary of the shadow is enlarged. To make its computed diameter agree best with the observed diameter, it is necessary to add $\frac{1}{60}$.

209. *Solar and lunar tables.*—In order to determine the circumstances of any particular eclipse, tables are needed which will give for that time the sun's and moon's hourly motions, their parallaxes, and their apparent semi-diameters. Such tables, of the most accurate kind, are published in the Nautical Almanac for each year, and several years in advance.

210. *The moon's relative orbit.*—The center of the earth's shadow moves in the ecliptic at the same rate as the sun, about 1° per day; while the moon moves in its orbit about 13° per day. To reduce these two motions to one, the relative orbit is substituted for the real one, in the following manner. Let NG (Fig. 58) be an arc of the ecliptic, Ng an arc of the moon's orbit, N the ascending node, A the place of the shadow's center, and a that of the moon's center, at the time of opposition. While A, in one hour, moves to A′, suppose a to move to g. Then A′g represents the distance and relative direction of the centers at the end of an hour after opposition. If gd be drawn equal and parallel to A′A, then Ad has the same length and direction as A′g. We may, therefore, suppose A, the center of the shadow, to have remained at rest, and a, the moon's center, to have moved to d in one hour; in which case, Fad would be the relative orbit, id (= hg) is the moon's hourly motion in latitude, and ai (= ah — AA′) is the difference of hourly motions in longitude.

Fig. 58.



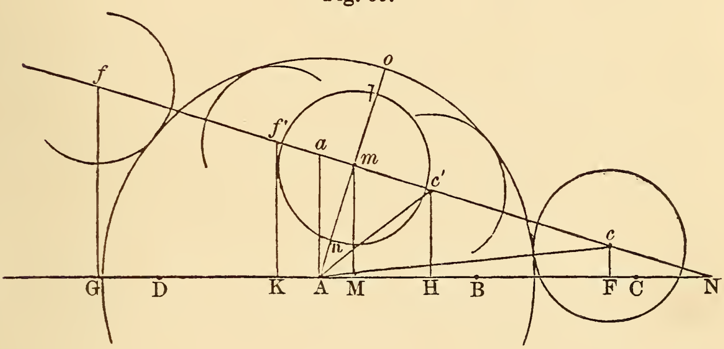
The inclination of the relative orbit to the ecliptic is found by the right-angled triangle dai, in which ai and di being known, the angle dai, or its equal, dFD, is computed.

The change from the true to the relative orbit is greatly exaggerated in the figure. If truly represented, ag would be 13 times as long as AA′.

211. *Times of beginning, middle, and end of a lunar eclipse, by projection.*—Let ND (Fig. 59) represent an arc of the ecliptic, and A the center of the shadow at opposition. With any convenient scale of equal parts, lay off from A the minutes of hourly motion of the moon from the sun—namely, AB, BC, AD, etc., and divide them into as small fractions of an hour as is desired. Then draw a circle with the radius Ao, equal to

the minutes in the semi-diameter of the shadow. Lay off A*a* perpendicular to CD, equal to the moon's latitude at opposition. Then *a* is the moon's center at that time. Through *a* draw N*f*, making N equal to the inclination of the relative orbit. Draw A*mo* perpendicular to N*f*. At the middle of the eclipse, the moon's center is at *m*, because A*m* bisects the chord of the circle. From *m* draw *m*M perpendicular to ND. The parts of hourly motion between M and A show how long before opposition the *middle* of the eclipse occurs.

Fig. 59.



Take a line equal to the sum of the semi-diameters of the shadow and the moon, place one end at A, and mark the points *c* and *f*, with the other end on the moon's path. With a radius equal to the semi-diameter of the moon, draw the circles around *c* and *f*, which will touch the shadow. The eclipse begins when the moon's center is at *c*, and ends when at *f*. Next draw the perpendiculars *c*F, *f*G, and we have on the scale of time the interval FA between the beginning and opposition, and AG between opposition and the end.

Finally, if the latitude is so small that the moon falls entirely into the shadow, making A*c'*, A*f'*, each equal to the difference of the two semi-diameters, mark the points *c'* and *f'* as before. Then the perpendiculars, *c'*H and *f'* K, mark the times of the beginning and end of the total eclipse.

212. *The middle of the eclipse, how related to the opposition.*—In the projection just described, N is the ascending

node, and the moon passes the node, N, before it reaches the opposition, a; in which case, the middle of the eclipse at m precedes the opposition at a. This is true at either node; the middle of the eclipse precedes opposition, if the passage of the node precedes it; but the middle is later than opposition, if the passage of the node is later.

213. *Times of beginning, middle, and end of a lunar eclipse by computation.*—The same results may be obtained with greater accuracy by trigonometry.

As Aa and Am are perpendicular respectively to ND and Nf, the angle aAm is equal to ANa, the angle of the relative orbit. The moon's latitude, Aa, being known, and the angle aAm, compute Am; then by Am and AmM ($= a$Am) find AM, and change it into time by the proportion, hourly motion in longitude of moon from shadow : MA : : 1 hour : time of passing over MA. Thus, the time of the middle of the eclipse is obtained.

Am and Ac being known, the angle mAc is calculated; which subtracted from mAM (complement of aAm) leaves cAN. Hence, in the triangle AcF, Ac and the angle cAF furnish FA, which, changed to time as before, determines the time when the eclipse begins. In the same manner, by the triangle Ac'H, the time of the beginning of the total eclipse is found. No additional calculation is necessary for the end; for the interval between the beginning and middle is equal to that between the middle and the end.

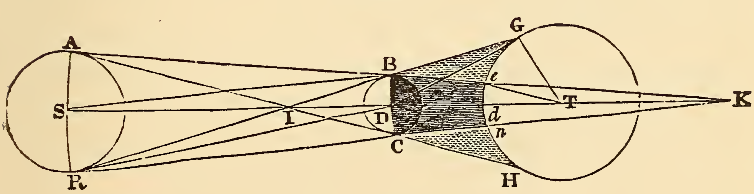
214. *Digits eclipsed.*—The magnitude of an eclipse is usually expressed in *digits*, or 12ths of the moon's diameter. The distance from n, the inner edge of the moon, to o, the edge of the shadow, is divided into parts, each equal to $\frac{1}{12}$ of nl. The number of such parts contained in no expresses the digits eclipsed. If the digits eclipsed equal or exceed 12, the eclipse is total.

215. *Eclipse of the sun.*—An eclipse of the sun is of a different character from an eclipse of the moon. When the moon is eclipsed, it is obscured by the earth's shadow falling on it

The moon itself is affected. But the sun is said to be eclipsed when the moon intervenes between it and the earth, and hides it from our view. The sun itself suffers no change, but we are placed in circumstances which prevent our seeing it. The phenomenon would more properly be called an *occultation* of the sun.

216. *Form and angle of the moon's shadow.*—The moon's shadow, like the earth's, is a cone, surrounded by a penumbra of infinite extent. Let AR (Fig. 60) be the sun, BC the moon, and K, the vertex of its conical shadow. The exterior angle SDR = DRK + DKR; ∴ DKR = SDR — DRK. Now, SDR is readily found, being the apparent semi-diameter of the sun as seen from the moon. It is larger than as seen from the earth, in the inverse ratio of distances, or as 387 : 386, nearly. The angle DRK is the sun's horizontal parallax at the moon. On account of distance, it is larger than at the earth, nearly in the ratio of 387 : 386; but on account of the moon's size, it is less in the ratio of their diameters, 2161 : 7912. The sun's horizontal parallax at the earth, when thus modified, gives the angle DRK. Therefore, DKR, the semi-angle of the moon's shadow, is found. Its mean value is 16′ 1″.6, about the same as the sun's apparent semi-diameter.

Fig. 60.



217. *Length of the moon's shadow.*—In the triangle DKC, right-angled at C,

$$\sin DKC : \text{rad} :: DC : DK,$$

the length of the moon's shadow. Its mean length is 231,690 miles, not quite sufficient to reach to the earth's surface.

When the moon is nearest to the earth, and the earth at the

same time is furthest from the sun, the shadow is long enough to reach about 14,500 miles beyond the earth's center.

218. *Greatest breadth of section on the earth.*—In the case just mentioned, if the shadow is directed toward the earth's center, its section at the surface is the greatest possible. To find its diameter en, compute the angle $e\mathrm{T}d$, thus:

$$e\mathrm{T} : \mathrm{TK} :: \sin e\mathrm{KT} : \sin \mathrm{T}e\mathrm{K},$$

and $e\mathrm{KT} + \mathrm{T}e\mathrm{K} = e\mathrm{T}d$. Then,

$$360° : e\mathrm{T}d :: \text{earth's circumference} : ed.$$

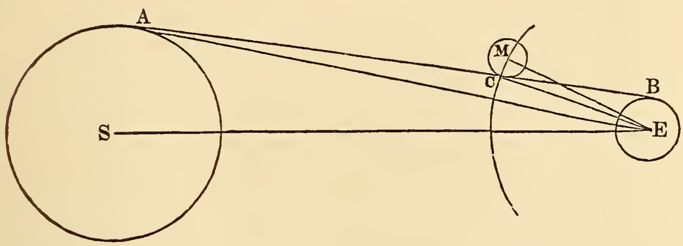
This, when greatest, is about 85 miles, and therefore the diameter of the section is 170 miles. Within this circle there is witnessed a total eclipse of the sun.

219. *The moon's penumbra, and its greatest section on the earth.*—The crossing tangents, ACH, RBG, etc., include the penumbra. Its semi-angle is BID, which is equal to IRD + IDR. But IRD is the sun's horizontal parallax at the moon, and IDR is the sun's apparent semi-diameter at the moon. Therefore, BID is known. To this add IGD, the moon's apparent semi-diameter, and the sum equals GDT. Hence, in the triangle GDT we have GT, TD, and the angle GDT, by which GTD is computed. From this, GH, the diameter of the penumbra on the earth, is obtained, as in the preceding article. Its greatest diameter is 4,500 miles.

220. *Solar ecliptic limit.*—The distance of the sun's center from the node, when the moon's penumbra at conjunction would only touch the earth in passing, is called the solar ecliptic limit. It is obtained by first finding the distance between the sun's and moon's centers at the given time. Let S (Fig. 61) be the sun's center, E the earth's, and M the moon's. It is obvious that the limit occurs when the moon's disk just touches AB, the extreme solar ray that meets the earth. The angular distance between the centers of the sun and moon at that time is the angle SEM. But SEM = SEA + AEC + CEM. SEA is the sun's semi-diameter $= \delta$. CEM is the moon's semi-diameter $= d$. The angle AEC (in the triangle EAC) = ECB — CAE.

But ECB is the moon's horizontal parallax $= P$; and CAE is the sun's horizontal parallax $= p$. Therefore, the distance between the centers, SEM $= \delta + d + P - p$; that is, the sum of the semi-diameters of the sun and moon, added to the difference of their parallaxes.

Fig. 61.



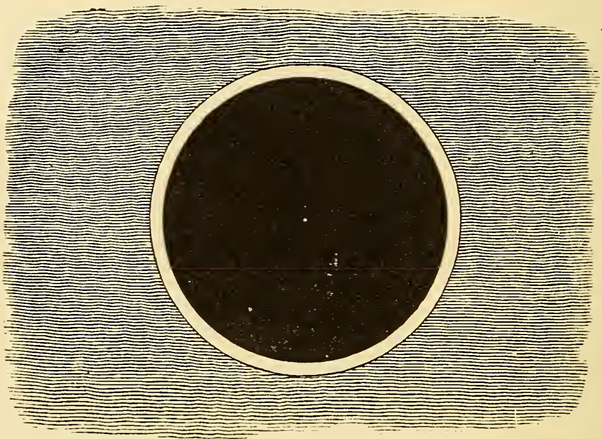
Representing this distance by CM (Fig. 57), CN is computed as in Art. 203. At the maximum, it is found to be 18° 36′; and beyond that, an eclipse is impossible. Its minimum value is 15° 20′; and within that, there cannot fail to be an eclipse.

221. *Magnitude of eclipse.*—If the eye of the observer were at the vertex of the total shadow of the moon, it is plain that the moon's disk would exactly cover the sun's. And as the moon appears to our unaided vision to be of the same size as the sun, this, of itself, shows that the cone of the shadow has a length sufficient to reach about to the earth, as proved (Art. 217). But the moon's semi-diameter is sometimes greater than the sun's, and sometimes less. When greater, the eclipse is *total* to all those places which fall within the section of the shadow as it crosses the earth. When less, the eclipse is *annular* to places lying sufficiently near the path of the axis of the shadow. It is called annular, because a *ring* of the sun's disk is seen about the moon (Fig. 62). An eclipse, whether total or annular, is *central* at all places where the axis of the shadow falls, or to which it points. If only the penumbra passes a place, the eclipse there is *partial*. The annular eclipse belongs to the class of partial eclipses.

If the total shadow reaches the earth at all, yet its section is small, compared with that of the penumbra (Arts. 218 and

219). Hence, at a given place, while partial solar eclipses occur frequently, probably one or two every year, a total eclipse is extremely rare, perhaps not one in a century.

Fig. 62.



It is possible for an eclipse to be annular to those places where it is seen in the morning or evening, and total to those in which it is seen near noon; for on the meridian, the moon appears about $\frac{1}{60}$ larger than at the horizon (Art. 165), and might cover the sun in one case, when it would not in the other. If an eclipse thus changes its magnitude from annular to total, and then to annular again, while crossing the earth, it results from the fact that the moon's shadow is *too long* to reach the nearest part of the earth's surface, and *not long enough* to reach its center.

222. *Velocity of the shadow.*—The hourly motion of the moon from the sun is about 30′. This arc equals 2,080 miles of absolute motion of the moon in its orbit. The shadow may be considered as having the same velocity as the moon. Therefore, the absolute velocity of the moon's shadow on the earth is 2,080 miles per hour, which is sufficient to carry it across the earth's disk in a little less than 4 hours. Relatively to the surface, the velocity is much less than this. At the equator, the

velocity of surface is about 1,040 miles per hour, one-half that of the shadow, and both motions are from west to east. Hence, at the equator, the shadow passes a place at the rate of about 1,040 miles per hour, when it falls perpendicularly. When inclined, as at morning and evening, it passes more swiftly, in the proportion of radius to the sine of obliquity. The relative motion is also greater as the latitude increases, on account of the slower motion of the surface. When an eclipse falls within polar circle, the shadow and the observer may possibly move in opposite directions, so that the relative motion would be the *sum*, instead of the *difference*, of the real motions.

223. *Duration of total and annular eclipses.*—The sun and moon differ so little in apparent size, and the velocity of the shadow is so great, that the duration of total and annular eclipses is necessarily short. It is seen by the preceding article that the rotation of the earth generally reduces the relative velocity; it therefore increases the duration. The greatest continuance of a total eclipse of the sun is about 8 minutes. An annular eclipse may continue more than 12 minutes.

224. *Number of solar and lunar eclipses.*—If an eclipse of the sun occurs in passing each node in a certain year, the lunar ecliptic limit is so small, that the moon may escape an eclipse at both the previous and the subsequent oppositions. In this case, there would be but *two* eclipses in a year, both solar. This is the least number.

If, however, a lunar eclipse occurs very near a node, the solar limit is so large, that there must be one, and there may be two solar eclipses at the preceding and following conjunctions. Thus, there may be as many as six eclipses while the sun passes the two nodes. Another one may possibly occur before twelve months have elapsed, in consequence of the backward motion of the nodes. Thus, the greatest number in a year is *seven*, of which five are of the sun, and two of the moon.

225. *Relative number of solar and lunar eclipses.*—Solar eclipses are more numerous than lunar, in the proportion of

their ecliptic limits—that is, nearly as 3 : 2. But, because one is really an eclipse, and the other an occultation, eclipses of the moon at a given place are more frequent than those of the sun. An eclipse of the moon is visible to all on the hemisphere nearest to it, without regard to locality. But an eclipse of the sun is not seen at a place, unless the moon's shadow falls at that place.

226. *Solar and lunar eclipses begin on opposite sides.*—As the moon moves toward the east much faster than the sun or the earth's shadow, we determine on which side of the body a solar or a lunar eclipse begins, by simply considering the motion of the moon. In a lunar eclipse, the moon overtakes the shadow of the earth, and, of course, its eastern limb enters the shadow first. Hence, a lunar eclipse always begins on the *east* side of the moon, and ends on the west side. But in a solar eclipse, the moon, in its eastward motion, overtakes the sun, and conceals its western limb first; so that a solar eclipse begins on the *west* side of the sun, and ends on the east side.

227. *The Saros.*—This name is given to the cycle of 18 years and 10 days, within which there is a return of the eclipses of preceding cycles, in the same order, and of nearly the same magnitude. The reason for this return of eclipses is, that the sun, moon, and node, return to very nearly the same relations to each other in the period just named.

The return of the moon to the sun (a lunation) occurs 223 times, and the return of the sun to the node (a synodical revolution of the node) occurs 19 times, in this period of 18 years and 10 or 11 days, the two periods differing less than 12 hours from each other. As the sun, moon, and node, do not resume their *exact* relation to each other, the series of eclipses in one cycle will vary a little from those of the preceding; and, therefore, after a number of cycles, their magnitude will become essentially changed, and at length, one after another, they will disappear from the cycle entirely.

This period was used by the Chaldeans for predicting the returns of eclipses, and by them called the *Saros.*

228. *Phenomena of a total eclipse of the sun.*—

1. The *corona.*—This is a luminous halo surrounding the moon when the sun is entirely hidden, and sometimes presents a radiated appearance, and extends from the moon's edge outward a distance equal to one-third of its diameter, fading gradually to the shade of the sky. It is concentric with the sun, rather than with the moon, and is thought to indicate an extensive solar atmosphere.

2. *Baily's beads.*—At the instant when the fine thread of the sun's edge is just appearing or disappearing, it is often divided up into a series of separate bright points. Being first noticed by Sir Francis Baily, they are known as Baily's beads. The appearance is by some attributed to the light of the sun's edge coming through between the mountain summits of the rough outline of the moon's disk. That they are not always seen, may arise from the fact that the limb in contact may, in some cases, be much less serrated by mountains than in others.

3. *Flame-colored protuberances.*—Another phenomenon, very variable in its aspect, consists of irregular projections, which appear here and there around the disk of the sun, after it is wholly in occultation. They are sometimes broad, and of small elevation; at others, they extend out nearly a tenth of the diameter of the sun—that is, to the height of 80,000 miles, and are often bent at a considerable angle. Occasionally, they are entirely detached from the disk. These flame-colored or rose-colored prominences, when first discovered, were not supposed to be sufficiently luminous to be seen except when the sun was wholly covered by the moon. But improved instruments have more recently rendered them visible at other times. They are found to consist mainly of red-hot hydrogen, thrown violently upward from the fiery surface of the sun.

A total eclipse of the sun is one of the most sublime and impressive phenomena of nature. The darkness is such, that the larger planets and stars appear; and yet it is surprisingly sudden in coming and going; for within a few seconds before and after the total darkness, the light is equal to that of hundreds of full moons. A chill is felt like that of night. It is not strange that people of barbarous countries are filled with con-

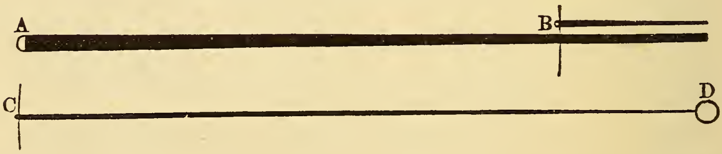
sternation and fear by the occurrence of a total eclipse of the sun. Appendix D.

229. *Eclipses at the moon.*—When we witness a solar eclipse, a spectator at the moon would notice only a small, dimly-defined circular shadow passing over the earth's disk. It would be a partial eclipse of the earth.

But when we see a total lunar eclipse, the phenomenon at the moon would be one of great interest, and of very strange appearance. A dim red light from all parts of the sun's disk is spread over the moon, being refracted thither by the earth's atmosphere (Art. 208). Hence, a spectator there would see the sun expanded out into a thin dull-red ring, surrounding the earth, and, therefore, having nearly four times the usual diameter of the sun's disk.

230. *True form of shadows.*—It is impossible, in ordinary diagrams, to present the shadows of the earth and moon in their true proportions. The distance of the sun is so very great, compared with its diameter, that the shadows are exceedingly slender, having a length about 110 times the diameter of the base. Fig. 63 is intended to exhibit them in their true forms. A is the earth, and B the moon, having just emerged from an eclipse. Only one-half of the whole length of the shadow of each is presented. Again, on another scale, C is the moon, and D the earth, on which its shadow is falling in a solar eclipse.

Fig. 63.



231. *Calculation of eclipses.*—Particular instructions are given in various works on practical astronomy for calculating all the circumstances of a solar or a lunar eclipse. Such instructions, with examples for illustration, may be found in Loomis's Practical Astronomy and Coffin's Solar and Lunar Eclipses.

CHAPTER XIII.

METHODS OF DETERMINING TERRESTRIAL LONGITUDE.

232. *Local time.*—Time is reckoned at every place from the moment when the sun crosses the meridian at either the upper or the lower culmination. This is called local time; for at the same absolute instant, the time thus reckoned at any place differs from that on every other meridian.

233. *Connection between longitude and local time.*—The earth turns uniformly on its axis toward the east through 15° every hour. Therefore, a place lying eastward of another will have the sun earlier on its meridian, and consequently, in respect to the hour of the day, will be in advance of the other at the rate of one hour for every 15°. Thus, to a place 15° east of Greenwich observatory, it is 1 o'clock P. M. when it is noon at Greenwich; and to a place 15° west of that meridian, it is 11 o'clock A. M. at the same instant. Hence, the difference of local time at any two places indicates their difference of longitude.

234. *Longitude by the chronometer.*—If a person leaves London with a chronometer accurately adjusted to Greenwich time, and travels eastward till he finds his own time slower than the local time of the place by 1h. 30m., then he knows the place to be 22° 30′ E. longitude. For $15° \times 1\frac{1}{2} = 22\frac{1}{2}°$. On the contrary, if he travels westward, and at length finds his time-piece at 6h. 44m., when the local time is 4h. 32m.—in other words, that his Greenwich time is 2h. 12m. too fast—then the longitude of the place is 33° W. In the same manner, the longitude of any two places may be compared with each other.

For the use of navigators, chronometers are made which run with very great accuracy, and may be relied on during long

voyages. There is always a probability, however, that a chronometer may change its rate somewhat, when it comes to be transported from place to place. It is therefore safer on long voyages to use several chronometers, and employ the mean of all their indications.

235. *Longitude by a lunar eclipse.*—In one respect, a lunar eclipse is very favorable for the comparison of longitudes. It is a distant phenomenon, seen at the same absolute instant by all. Hence, any difference of time in the observations at different places is entirely due to difference of longitude.

But in another respect, it is quite unfitted for the purpose. On account of the penumbra, there is no definite edge to the shadow which passes over the moon's disk, and consequently there is great uncertainty as to the time of beginning or end of the eclipse. This method is but little depended on for accurate results.

236. *Longitude by a solar eclipse.*—In both the above particulars, a solar eclipse differs from a lunar. It is not an event at a distance, seen at once by all, but on the earth's surface, happening to one place at one instant, and to another place at another. The time of beginning or end of a solar eclipse depends on the position of the observer.

On the other hand, the phenomenon is very definite, and the moments of immersion and emersion of the sun's limb can be quite accurately fixed by observation.

To compare longitudes by a solar eclipse, the observations made on the beginning and end at a given place are used as means of calculating the time of conjunction—that is, the time when the sun and moon are in the same secondary of the ecliptic. But that event occurs at a certain absolute instant. This computation being made for each place, the time of conjunction ought to be exactly the same, so that the difference in the results is wholly due to a difference in the longitude of the places. This method of obtaining the longitude of a place is accurate, but laborious.

Occultations of *stars* by the moon are much more frequent than the occultation of the *sun;* and these are phenomena of

the same general character, and may be used in the same way for finding the longitude of a place.

237. *Longitude by eclipses of Jupiter's satellites.*—The satellites of Jupiter fall into the shadow of that planet, as the moon does into the shadow of the earth. Every such eclipse occurs at a certain time; and all who see it, see it at the same instant. Hence, these eclipses are favorable for determining longitudes. Moreover, they are occurring every day, while eclipses of the sun and moon are rare.

But, on account of the penumbra of the planet, and the considerable diameter of the satellites, they disappear and reappear gradually. There is difficulty, therefore, in observing accurately the beginning and end of these eclipses. In order to obtain the best results, the telescopes used by different observers ought to be alike in aperture and power.

238. *Longitude by the lunar method.*—This is a method particularly useful to navigators, because the observations are made by the sextant. It consists in measuring the angular distance between the moon and some conspicuous heavenly body, as the sun, or a large planet or star, and then correcting the observation for parallax and refraction, so as to have the true distance between the bodies, as seen from the center of the earth. The observer must also note the local time when this measurement is made.

Having with him the Nautical Almanac, in which the distances, as seen from the earth's center, are predicted for every day and hour of Greenwich time, he looks for the Greenwich time at which the distance agrees with the distance as he has obtained it. The absolute time is the same; hence, the *difference* of time shows his longitude from Greenwich.

The bodies, whose angular distances from the moon the Nautical Almanac gives for every three hours, with proportional numbers for interpolation, are the sun, Venus, Mars, Jupiter, Saturn, and nine bright fixed stars.

239. *Longitude by the telegraph.*—Since the invention of the magnetic telegraph, it has been employed to determine the

differences of longitude between fixed stations on land with a precision which was before altogether unattainable. Suppose two stations to be connected by the telegraphic line, and that there is at each a clock keeping the local time. The observers agree beforehand at what time, by his own clock, the one at the most easterly station shall commence giving signals; and also at what time the other shall commence giving another series according to *his* clock. The interval between successive signals is also previously determined. When the moment arrives, the first observer strikes the telegraphic key at the exact beat of the clock, and the second observer records the *time* of the signal as shown by his own clock; and thus they continue to do till the full series is recorded. The second observer then commences sending signals, which are in like manner recorded by the first. The velocity of the electric current is so great, that the *absolute* time of making a signal at one station, and of perceiving it at the other, may be considered identical; so that the difference which is indicated by the two clocks in each case is wholly due to difference of longitude. Still greater precision is attained by causing the signal key at each station to record its own movement on the line of second-marks made by the clock at the other station (Art. 46).

240. *Velocity of the electric current.*—The method just described is susceptible of such accuracy, that it has led to the discovery of the velocity of the current. For, if the moment of its arrival at the distant station is not identical with that of the signal given, it will indicate a difference of longitude *less* than the true difference when sent westward, but *greater* than the true difference when sent eastward. By this discrepancy, if it is appreciable, the velocity of the current becomes known. It is found to be about 16,000 miles per second.

241. *Change of days in circumnavigating the earth.*—While a person travels westward, he lengthens his days by one hour for every 15°, or 4 minutes for every degree, since he moves along with the apparent diurnal motion of the sun. In traveling eastward, on the contrary, he shortens the days at the same

rate, by moving in opposition to the sun's daily progress. If we suppose him to go westward entirely round the earth to the same meridian again, whether he takes a longer or a shorter time for the journey, he will lengthen the individual days sufficiently to make the whole number just *one day less* than if he had remained where he was. The 5th of a month is to him the 4th; and Tuesday, according to his reckoning, is Monday. The reason is obvious; for during his journey, the earth has made a certain number of diurnal revolutions from west to east; but he, by going round from east to west, has, in respect to himself, diminished that number by one.

All this is exactly reversed when one goes round the globe from west to east. He gains just a day by making all the days of his travel a little shorter. It is plain that he makes one more diurnal revolution from west to east than the earth does.

Of course, if these two individuals meet at their place of starting, they differ from each other just *two* days in their reckoning.

242. *Ambiguity as to days among the islands of the Pacific Ocean.*—If an island in the Pacific were settled by navigators, who had gone westward around Cape Horn, and also by others, who had sailed eastward around the Cape of Good Hope, the reckoning of these two parties would differ by one day. To the former, a day will be the first of a month when it is the 2d to the latter. It is, in fact, true that there are islands lying contiguous to each other which have this difference of reckoning.

If inhabited land extended entirely round the earth, it would be necessary to fix arbitrarily on some meridian on which the change of day should be made. For it is impossible that the reckoning of days should go on unbroken around the earth. The arbitrary meridian would separate between places which differ a day from each other; so that, on the west side of it, the time is one day later, both in the month and the week, than on the east side.

CHAPTER XIV.

THE TIDES.

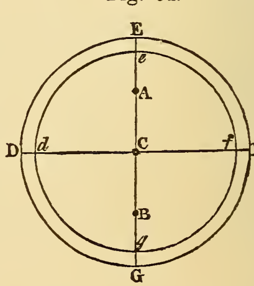
243. *Definitions.*—The *tides* are the daily rising and falling of the waters of the ocean. When the water, in this daily oscillation, has reached its highest point, it is called *high-water;* at its lowest point, it is called *low-water.* While the water is rising, it is called *flood;* and while falling, *ebb.*

A *lunar day* is the time between two successive culminations of the moon. Its length is about 24h. 52m., being nearly an hour longer than a solar day on account of the rapid eastward motion of the moon. The tides make their revolutions within the lunar day.

Twice in a lunation high-water is at a maximum, and twice it is at a minimum; the former are called *spring tides*, the latter, *neap tides.* The spring tides occur near the time of syzygies, the neap tides near the time of quadratures.

244. *Opposite tides.*—There are two tide-waves on opposite sides of the globe, moving around it from east to west, and arriving at any place at intervals, whose mean value is 12h. 26m., or half a lunar day. Since the mean diurnal motion of each of the two opposite tides is the same as that of the moon, the action of the moon must be regarded as the principal cause of the tides.

Fig. 64.



245. *Form of the water acted on by the moon.*—If the earth were covered with water, and no force were exerted except gravitation toward the earth itself, its form would be exactly spherical, as represented in Fig. 64. But if a distant body, as the moon, should also attract it, the sphere would be changed into a *prolate spheroid*—that is, into a form produced by revolving an ellipse about its major axis. Let the moon be in

the direction of CE produced, and suppose the center of gravity of the nearer half of the water, DEF, to be at A, and that of the remote half at B, while the center of the earth, as a whole, is at C. Since A is more attracted than C, and C more than B, the form of equilibrium must be disturbed, and some of the water will flow toward E, and other parts toward G, till the particles are in equilibrio between their unequal tendencies to the moon, and their gravity on the inclined surface of the spheroid. E and G are the highest points of the spheroid, and all points on the circle DF (perpendicular to EG) are the lowest. Every section through EG is an ellipse, whose major axis is EG, and whose minor axis is equal to DF. The ellipticity of the section will obviously depend not only on the *strength* of the moon's attraction, but also on the *difference* between the attractions on the nearer and remoter parts.

In the case of the earth and moon, it is computed that the major axis would exceed the minor by 5 feet—that is, the tides would be only $2\frac{1}{2}$ feet high, and on opposite sides of the earth, one directed *toward* the moon, the other *from* it. The tide on the side nearest the moon is sometimes called the *direct* tide; the one on the remote side, the *opposite* tide.

246. *Tides by the sun.*—The same kind of effect is produced by the sun as by the moon. But the distance of the sun is so great, that though it attracts the earth more than the moon does, yet the difference of its attractions on the several parts is less. The power of the moon to raise a tide is to that of the sun about as 5 to 2.

247. *Joint action of the sun and moon.*—At the time of conjunction, the moon and sun attract in the same direction, and therefore the tides are equal to the sum of the lunar and solar tides. The same is true at opposition, because each body produces two tides at once; and the direct lunar tide coincides with the opposite solar tide, and *vice versa*. These are the spring tides which occur at the syzygies.

At quadratures, each body raises a tide at the expense of that raised by the other. For if the moon is in the direction of EG produced (Fig. 64), it causes high-water at E and G,

and low-water at D and F. And if the sun is in the direction of DF produced, it causes high-water at D and F, and low-water at E and G. As the lunar tides are the highest, E and G are the neap tides, made less by this action of the sun, than if the moon had acted alone.

248. *Effect of the inertia of water.*—If the moon and earth were at rest, the tides would be directed exactly to and from the moon. But while the waters are flowing toward these points, the moon, by the diurnal motion, passes westward, and causes them to change the places at which they tend to accumulate. Thus, even if the wave were unchecked by the shores of continents and islands, the summit would be two or three hours behind the moon in passing a given meridian.

249. *Diurnal inequality.*—At a given place, the two tides which follow the culmination of the moon will vary in height, according to the relation between the latitude of the place and the moon's declination. If the moon, M (Fig. 65), is on the equator, it is clear that the tides on the equator, EQ, are greatest, and that in other places they are less, as the latitude is greater. But the two successive tides at any place are equal; for, by the rotation on NS, the tide at B in 12½ hours will come round to A, and be equal to the tide now there. The same is true of the tides C and D, or F and G. Hence, when the moon has no declination, there is no diurnal inequality.

Fig. 65.

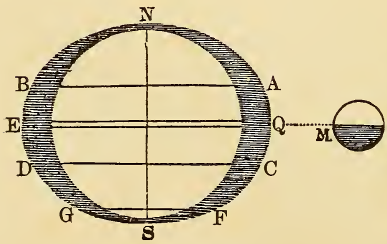
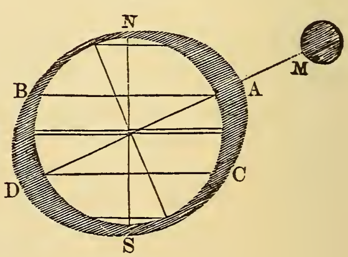


Fig. 66.



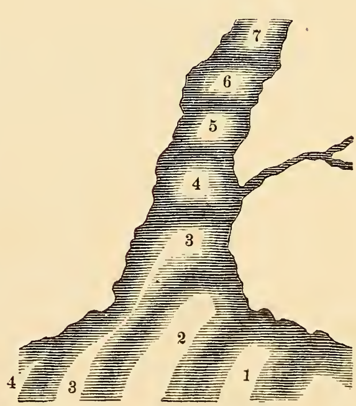
But suppose the moon has a northern declination, as in Fig 66. Then the highest points of the tide are at A in north lat

itude, and D in south. At A, where the direct tide is large, the opposite tide now at B will arrive in 12½ hours, and will be small. But at C, this is reversed; the direct tide is small, and the opposite one (now at D, and arriving at C 12½ hours later), is large. Therefore, when the declination and the latitude are both north, or both south, the direct tide—that is, the tide which first succeeds the upper culmination of the moon—is larger than the opposite tide; but if one is north, and the other south, the direct tide is smaller than the opposite tide. This difference in the height of the two successive tides is called the diurnal inequality.

250. *Change of direction and velocity caused by coasts.*—The tide-wave, which would move regularly westward around the earth, if it were wholly covered by deep water, is exceedingly broken up and changed, both in direction and velocity, by coasts and shoals. Its general direction is westward; but as it can pass the continents only at their southern extremities, it bears to the northwest, and then to the north, in the Atlantic and Pacific oceans; and when it enters seas or channels, it usually bends its course in the direction of their length.

251. *Cotidal lines.*—These are lines drawn on a chart of the oceans, showing the position of the summit of the tide-wave for each hour of a day. Such a system of lines expresses to the eye the direction and velocity of the tide at all places. Thus, on the open ocean, the figures 1, 2, 3, 4 (Fig. 67) show the situation of one and the same tide-wave at those hours, respectively. And in the channel which extends northward, the wave, having separated from the ocean tide, advances northward, and occupies the places marked at the hours indicated. The wave advances most rap

Fig. 67.



idly in the deepest water. Hence, the front is generally convex as in Fig. 67, since it moves fastest in the central part, where the water is deepest. For this reason, also, the tide may occupy as long a time in running through a long channel of shallow water as in advancing half round the earth. The greatest velocity of the tide in the deep, open ocean, is near 1,000 miles per hour. Some channels are affected by tides entering at both extremities. For example, the German Ocean and English Channel receive the Atlantic tide both at the north and at the south end. As a consequence, the tide system is doubled, causing, at some points, four tides per day.

252. *Modification in the height of the tide caused by coasts.*—The relation of coast lines to each other also very much affects the *height* of the tide at particular places. When the tide directly enters a broad-mouthed bay, it grows higher as the bay contracts in breadth; and at the head of the bay, there is usually found the greatest height of all. One of the most remarkable examples is the Bay of Fundy. The western extremity of the Atlantic tide-wave, after entering this bay, is gradually contracted by the shores as it advances, till, at the head of the bay, it sometimes rises to 70 feet.

The height of the tide on the coast is generally greater than in the open ocean, owing to the effect of shoal water. The most advanced part of the wave moves slower than the hinder portion; so that the cross-section of the ridge becomes shorter, and therefore higher, as the depth of water diminishes.

The mean height of the spring tides at any place is called the *unit of altitude* for that place.

253. *Establishment of a port.*—This phrase signifies the mean interval between the culmination of the moon and the arrival of the tide at a given place. At every meridian, the tide arrives later than the body which causes it; but the delay varies exceedingly at different localities, on account of shoal water, direction and length of channel, etc. Even at the same place, the delay during a lunation varies according as the small solar tide precedes or follows the large lunar one; for the summit lies between them. It is the *mean* interval at

a given port, which is called the establishment of that port.

254. *Tides of lakes and inland seas.*—In general, the tides of lakes and inland seas are scarcely perceptible. The reason is, their extent is so small, that all parts are to be considered as almost equidistant from the moon. There is little opportunity for water to be attracted from the more distant to the nearer part. The largest North American lakes have tides but an inch or two in height. In the Mediterranean, however, which derives no tide from the ocean, the tide-wave reaches $1\frac{1}{2}$ or 2 feet.

255. *Tides modified by the sun's and moon's change of distance.*—The difference of the moon's attraction on the several parts of the earth is greatest when the moon is nearest, and least when it is most distant. The same is true of the sun. Hence, the tides of each month have a periodical increase and decrease as the moon passes through its perigee and apogee. They have a like, though much smaller, change each year, at the perihelion and aphelion of the earth's orbit. By the revolution of the apsides of the moon's orbit, these maxima and minima will alternately coincide once in 9 years. Combining these changes with those at syzygy and quadrature, the height of the greatest possible spring tide, to that of the least possible neap tide, is as 10 to 3.

CHAPTER XV.

THE PLANETS.—TABULAR STATEMENTS.—MERCURY.—VENUS MARS.

256. *Names and classification of the planets.*—The *planets* are solid spherical bodies revolving about the sun in orbits which are nearly circular. The name "planet" signifies a *wanderer*, and was given to these bodies because they continually change their places among the fixed stars, generally moving from west to east, but sometimes from east to west. These apparently irregular motions are fully explained by our own annual motion, the earth on which we live being one of the planets.

The planets are naturally arranged in three classes.

1. Four *small* planets near the sun, of which the earth is the largest—namely, *Mercury*, *Venus*, *Earth*, *Mars.*

2. The *planetoids*, an indefinite number of bodies, too small to be measured with certainty, and occupying a ring outside of the first class. They are also called *asteroids*, and *minor planets.*

3. Four *large* planets, moving outside of the ring of planetoids, widely separated from each other, and at vast distances from the sun. These are *Jupiter*, *Saturn*, *Uranus*, *Neptune.*

Two planets of the first class, Mercury and Venus, revolve in orbits within the earth's orbit. These are called *inferior* planets, being *lower* down in the solar system than the earth is. All the others, including the planetoids, are called *superior* planets; because, in relation to the sun, the great center of attraction, they are *higher* than the earth, and revolve in orbits exterior to the earth's orbit. Appendix F.

257. *Satellites.* There is another class of spherical bodies, holding a subordinate place in the solar system, since they revolve around the planets as centers. These are called *satellites.* The moon, already described in Chapter X., is a satellite

of the earth. They are distributed as follows : the Earth has 1; Mars, 2; Jupiter, 4; Saturn, 8; Uranus, 4; Neptune, 1. Mercury and Venus have no satellites.

The satellites are also called *secondary* planets: and the planets, in distinction from them, *primary* planets.

258. *Distances of the planets from the sun.*—The following table presents the mean distances of the planets from the sun in millions of miles, and also their relative distances, the earth's being called 1.

I.

	Mean Distances.	Relative Distances.
Mercury	36,000,000	0.39
Venus	67,000,000	0.72
Earth	92,000,000	1.00
Mars..............	141,000,000	1.52
Planetoids........	250,000,000	2.67
Jupiter...........	481,000,000	5.20
Saturn	881,000,000	9.54
Uranus...........	1772,000,000	19.18
Neptune	2775,000,000	30.05

It appears by this table, that the remotest planet is 77 times as far from the sun as the nearest. Hence it is that orreries, unless of inconvenient size, always fail of truly representing the planetary distances. The same is generally true of diagrams.

259. *Periodic times of the planets.*—The following table contains the length of the sidereal revolutions in months and years, which is the most convenient form for the memory; their length in days and decimals, for calculations; their mean daily motion; and the time of their diurnal rotations, so far as known, in hours and decimals.

II.

	Sidereal Revolution.	Sidereal Revolution in Days.	Mean Daily Motion.	Diurnal Rotation.
Mercury...	3 months.	87.969	4° 5′ 32″.5	24.09 h.
Venus.....	$7\frac{1}{2}$ "	224.701	1° 36′ 7″.7	23.35 "
Earth	1 year.	365.256	0° 59′ 8″.3	23.93 "
Mars	2 "	686.980	0° 31′ 26″.5	24.66 "
Planetoids .	$4\frac{1}{2}$ "			
Jupiter....	12 "	4332.554	0° 4′ 59″.1	9.92 "
Saturn	29 "	10759.104	0° 2′ 0″.5	10.24 "
Uranus....	84 "	30686.246	0° 0′ 42″.2	
Neptune ..	165 "	60228.072	0° 0′ 21″.5	

It will be found, by comparing the squares of any two periods in Table II, and the cubes of the corresponding distances in Table I, that their ratios are nearly the same; and this should be true according to Kepler's third law (Art. 119). Thus, for Neptune and the earth, $30^3 : 1^3 = 27,000$; and $165^2 : 1^2 = 27,225$. So also, while Neptune is 77 times as far from the sun as Mercury is, its period of revolution is 685 times as long. For $77^3 : 1^3 :: 685^2 : 1^2$, nearly.

Since the periods increase more rapidly than the radii of the orbits, the *velocities* of the planets must become less, the further they are from the sun. The distance described by Mercury in a day is nearly nine times that which Neptune passes over in the same time.

III

	Diameters.	Apparent Diameters.	Volumes.
Sun............	860,000	32′ 4″	1,295,000.000
Mercury........	2,992	0′ 7″	0.054
Venus..........	7,660	17″	0.880
Earth	7,918		1.000
Mars...........	4,211	9″	0.248
Jupiter.........	86,000	37″	1,350.000
Saturn..........	70,500	16″	689.000
Uranus.........	31,700	4″	75.000
Neptune........	34,500	3″	102.000

260. *Magnitudes of the planets.*—Table III gives the diameters of the sun and planets in miles, their mean apparent diameters, and their volumes compared with the earth.

In comparing the numbers of this table, it is noticeable that in general the planets diminish in size in each direction from the planetoids. If we suppose Mars to be placed between Mercury and Venus, and Uranus and Neptune to change places with each other, this would be strictly true.

Observe also that the diameters of the large planets beyond the planetoids are from eight to eleven times as large respectively as those of the small ones within that group. Thus,

Diam. of Jupiter	:	that of the earth	::	11 : 1,	nearly.
" Saturn	:	" Venus	::	9 : 1,	"
" Neptune	:	" Mars	::	8 : 1,	"
" Uranus	:	" Mercury	::	11 : 1,	"
and the sum	:	the sum	::	10 : 1,	"

Another remarkable fact appears on comparing the diameters in Table III, and the times of diurnal rotation in Table II. The four small planets all rotate in periods of about 24 hours. But the large planets, so far as known, revolve in about 10 hours. Hence, the equatorial velocity of rotation is far greater on the large than on the small planets. That on Jupiter, for example, is 27 times as great as that on the earth.

The dimensions of the planetoids are not given in the table, being too small for measurement. One or two of the largest are thought to be from 100 to 200 miles in diameter.

IV.

	Masses.	Density.	Specific Gravity.
Sun............	326,800.000	0.25	1 4
Mercury........	0.065	1.21	6.8
Venus..........	0.769	0.85	5.2
Earth..........	1.000	1.00	5.5
Mars...........	0.111	0.73	4.2
Jupiter........	311.953	0.24	1.3
Saturn.........	93.329	0.13	0.8
Uranus.........	14.460	0.22	1.3
Neptune........	16.862	0.20	0.9

261. *Masses and densities of the planets.*—Table IV exhibits the masses and densities of the sun and planets, the earth being called 1; also their specific gravities.

It appears from table IV, that the small planets are much more dense than the large planets and the sun.

262. *The sun and planets compared.*—By Table III, we see that the sun has 10 times the diameter, and 1,000 times the volume of Jupiter, the largest planet in the system. Table IV shows that the mass of the sun is also more than 1,000 times as great as that of Jupiter, and 700 times greater than the united masses of all the planets. Its attraction mainly controls the movements of all the planets, satellites, and comets. Hence, these bodies describe their various paths about it, scarcely disturbing it from a state of rest. For this reason, this system of bodies is called the *solar system.*

263. *Diameters of planets, and their distances from the sun.*—One of the most remarkable facts relating to the planets is brought to view in comparing the distances in Table I with the diameters in Table III. While the diameters of the planets are only a few thousands of miles, their distances from the sun are many millions. The diameter of Neptune's orbit is more than 20,000 times the diameters of all the planets added together. To attempt to represent both the distances and magnitudes of the planets in their proportions, by an orrery or diagram, is out of the question.

264. *Directions of the planetary motions.**—It has been

* It is desirable that the student should be able to recognize the planets, and become familiar with their motions. Some aid can be had by the use of the common almanacs. The "American Ephemeris and Nautical Almanac," published annually at Washington, can be purchased for one dollar by applying to the Bureau of Navigation. This gives the exact places of the sun, moon, and planets for each day of the year.

The orrery, heliotellus, and lunatellus are instruments that explain their motions. The planisphere is an inexpensive instrument that shows the places of several hundred of the more conspicuous fixed stars. It can be readily adjusted for any hour of the night. The astral lantern is a similar device, of larger size, for exhibiting maps of the stars on the illuminated sides of a cubical box. Appendix M.

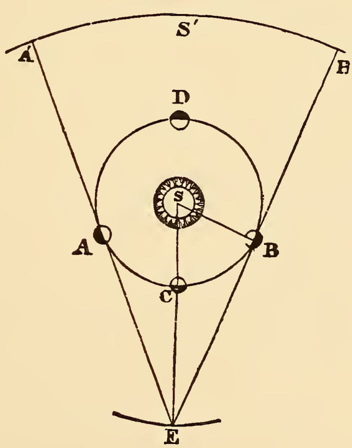
stated in preceding chapters that all the motions of the sun, earth, and moon are from west to east. The same thing is true, in general, of all the planets and satellites; and in nearly every case the inclination to the ecliptic is very small. The only exceptions are found in the satellites of Uranus and Neptune, whose planes of revolution are nearly perpendicular to the ecliptic, and the motion in them from east to west. All the planetoids yet discovered revolve from west to east, though the orbit of one of them has an inclination as large as 34°.

Since the motions in the solar system are so generally from west to east, this is regarded as *direct* motion; and any motions, real or apparent, which are from east to west, are called *retrograde.*

MERCURY.

265. *Tabular statements.*—Mean distance from the sun, 35,761,000 miles; periodic time, 3 months; diameter, 2,993 miles; diurnal rotation, 24.09 hours; specific gravity, 6.8.

Fig. 68.

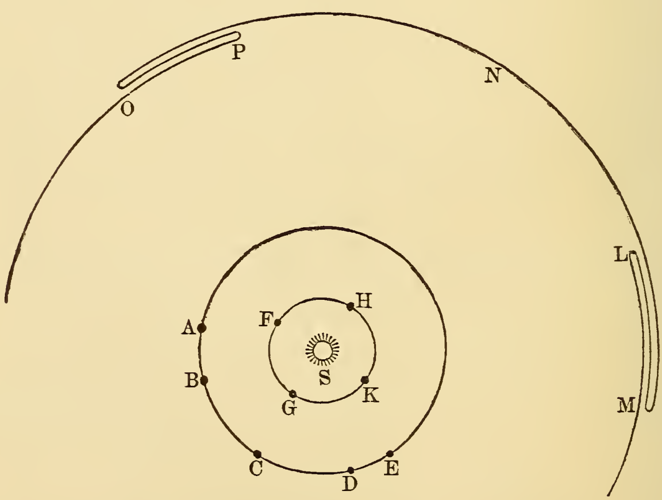


266. *Apparent motions.*—Mercury is an inferior planet, whose orbit is far within the earth's; for it is seen alternately east and west of the sun, and never more than 29° from it. Let E (Fig. 68) be the earth, supposed, for the present, to be at

rest; the circle ABD, the orbit of Mercury; S, the sun; and B′A′, the sky, on which the bodies are seen projected. When Mercury is at B, it is seen at B′; as it passes through D to A, it appears to advance to A′; as it is now coming toward the earth, it seems to be stationary at A′; then from A through C to B, it appears to retrograde from A′ to B′, where it is again stationary, as it moves away from us. Since the sun appears at S′, the planet passes by it, both when advancing and when retrograding.

When the planet is at D and C, it is in conjunction with the sun; at C, between the earth and sun, it is said to be in the *inferior* conjunction; at D, in *superior* conjunction. B and A are called the points of *greatest elongation.* At superior conjunction, the motion of Mercury appears to be forward; at the inferior conjunction, backward; and if the earth were at rest, as we are now supposing, the planet would appear stationary at the points of greatest elongation.

Fig. 69.



267. *The motions of Mercury as modified by the earth's motion.*—To simplify the case, it was supposed, in the preceding article, that the earth is at rest. But the earth moves in

nearly the same direction as Mercury, making about one revolution while Mercury makes four (Table II). The effect is to lengthen the arc of apparent advance, and shorten that of retrogradation. Thus, let the earth be at A (Fig. 69), when Mercury is at F; then it will appear in the sky at L. While the earth is advancing to B, Mercury passes the inferior conjunction, and arrives at G, and appears at M, having moved apparently backward from L to M. As the earth moves to C, Mercury describes GKH, and is at superior conjunction N. Again, while the earth moves to D, Mercury passes round to G, still advancing in the sky to O. But while the earth describes DE, Mercury again passes the inferior conjunction from G to K, and apparently retrogrades from O to P; after which, it begins once more to advance. Thus, by the earth's motion, the planet is made to retrograde through a shorter arc, and to advance through a longer one, than if the earth were at rest.

268. *Stationary points.*—If the earth were at rest, as supposed in Fig. 68, the points where the planet would appear stationary, in relation to the stars, would be A and B, at which tangents drawn from the earth would meet the orbit. But the earth's motion removes the apparently stationary points a little way toward the inferior conjunction. For, in order to appear stationary, the advance which the earth's motion causes, must be just neutralized by the retrogradation of Mercury. This planet appears stationary, when its elongation from the sun is 15° or 20°, according as it is nearer the perihelion or the aphelion.

269. *The synodical period of Mercury.*—This is the time in which it goes from a conjunction to the next conjunction of the same kind—that is, describes one revolution relatively to the earth instead of a star.

The sidereal period having been obtained by observing the planet's return to its node, the synodical period can be computed from it by using the relative motions of Mercury and the earth, just as we find the time in which the minute-hand of a watch will overtake the hour-hand. The synodical period of Mercury can also be found independently, by means of transits

across the sun's disk. The synodical period of Mercury is 116 days, which is nearly a month longer than its sidereal period.

270. *Form and position of Mercury's orbit.*—The orbit of Mercury is more eccentric, and more inclined to the ecliptic than that of any other of the eight planets. While the eccentricity of the earth's orbit is only $\frac{1}{60}$, that of Mercury is nearly $\frac{1}{5}$. Yet this renders the minor only $\frac{1}{50}$ shorter than the major axis; so that the form of the most eccentric of the planetary orbits, if correctly drawn, would appear to the eye to be a circle.

The inclination of Mercury's orbit to the plane of the ecliptic is 7°.

271. *Phases of Mercury.*—At the inferior conjunction, C (Fig. 68), the unilluminated side of Mercury is turned toward the earth, so that, like the new moon, it is invisible. At the superior conjunction, D, its illuminated side is toward us, and it is full. At A or B, where the ray AS, and our line of vision, AE, are at right angles, the phase is a semicircle. On the arc ACB occur the crescent phases: on BDA, the gibbous phases.

272. *Point of greatest brightness.*—Mercury is not brightest when full, because it is then too far distant. It is not brightest when nearest, because its dark side is toward us. Nor is it brightest at the place of greatest elongation; but beyond it, toward the superior conjunction, when about 22° from the sun. Its apparent diameter, when nearest the earth, and when most distant from it, is as $2\frac{1}{3}$ to 1.

273. *Transits of Mercury.*—As Mercury, at the inferior conjunction, passes nearly between the earth and sun, it may possibly come exactly in a line with them, and thus be seen as a black round spot going across the sun's disk. This phenomenon is called a *transit* of Mercury. If the plane of its orbit were coincident with that of the ecliptic, a transit would obviously occur at every inferior conjunction. Since the angle between the two planes is 7°, the planet can not be seen on

the disk, unless near the node, for its perpendicular distance from the ecliptic must be less than the sun's apparent semi-diameter—that is, less than 16′. By a simple calculation, like that in Art. 203, it is found that the limit of transit for Mercury is 2° 10′.

274. *Node months for Mercury.*—The nodes of Mercury's orbit lie in that part of the heavens which the sun passes through in May and November. Therefore, a transit of that planet can occur only in those months. More transits happen in November than in May, because the planet is nearer perihelion in November, and therefore more likely to be projected on the sun's disk. After the lapse of ages, the months will change, on account of the slow retrograde motion of the nodes.

275. *Intervals between transits.*—While the earth makes 13 revolutions from a node to the same node again, Mercury makes 54 revolutions, very nearly. Hence, in 13 years after a transit, the two bodies will return so nearly to the same relations to the node, that another transit is likely to occur. The least interval between transits at the same node is 7 years, in which time Mercury makes very nearly 29 revolutions. As these are both odd numbers, the period may be halved, and a transit may occur in 3½ years at the other node. This is the shortest interval. The transits of Mercury in the last half of the present century are the following: November 11, 1861; November 4, 1868; May 6, 1878; November 7, 1881; May 9, 1891; November 10, 1894.

VENUS.

276. *Tabular statements.*—Mean distance from the sun, 66,822,000 miles; periodic time, 7½ months; diameter, 7,660 miles; diurnal rotation, 23.35 hours; specific gravity, 4.8.

277. *Apparent motions.*—Like Mercury, Venus appears to pass back and forth by the sun, reaching a distance of 47° at its greatest elongation. This proves it to be an inferior planet, between Mercury and the earth. Its sidereal period approaches so near to that of the earth, that its synodic period is length

ened to nearly $1\frac{2}{3}$ years. Hence, after making an apparent retrograde motion, as LM (Fig. 69), it advances twice and two-thirds round the heavens before it commences the next retrograde arc, OP.

278. *Phases and brightness of Venus.*—Venus passes through the same changes of phase as Mercury. But its apparent diameter, when the crescent phase is narrowest, is more than 6 times as great as when at full. For its distance from us, in the former case, is 92,000,000 − 67,000,000 = 25,000,000 miles; and in the latter, it is 92,000,000 + 67,000,000 = 159,000,000 miles, a distance more than six times as great as the other.

Venus is the brightest of the planets, and has been known from ancient times as the morning and evening star, according as it is west of the sun, or east of it.

The place of greatest brightness for Venus is when about 40° from the sun, between the point of greatest elongation and the inferior conjunction. In this situation, it is frequently visible all day.

279. *Transits of Venus.*—The orbit of Venus is inclined to the ecliptic about $3\frac{1}{2}$ degrees. The sun passes its nodes in June and December; therefore, the transits of that planet occur in those months.

Venus makes 13 revolutions in very nearly the same time in which the earth makes 8. Hence, a transit of Venus at either node is usually preceded or followed by another at the same node, at an interval of 8 years. But this interval can not be halved, as in the case of Mercury (Art. 275), to find the time of a transit at the other node; because, 8 being an even, and 13 an odd number, there would, in 4 revolutions of the earth, be $6\frac{1}{2}$ revolutions of Venus, which would bring the two planets on opposite sides of the sun.

The interval of 235 years is much more exactly measured by 382 revolutions of Venus. Therefore, after a transit, there is almost a certainty of another at the same node in 235 years. But, for the same reason as before, the middle of this interval can not be taken as the date of a transit at the other node.

The smaller intervals must be obtained by using the period of 227 years, which is 8 years less than 235 years.

In 227 years, there are 369 revolutions of Venus within 1½ days. Hence, transits are very likely to occur at the same node at intervals of 227 years. And at the middle of this interval, there will probably be a transit at the other node, since 113½ revolutions of the earth, and 184½ of Venus, bring both bodies to the opposite side of the heavens. This interval of 113½ years may be increased or diminished by 8, to furnish two other intervals. Hence, the ordinary intervals are 8, 105½, 113½, and 121½ years, as may be seen in the following series of transits from 1518 to 2004:

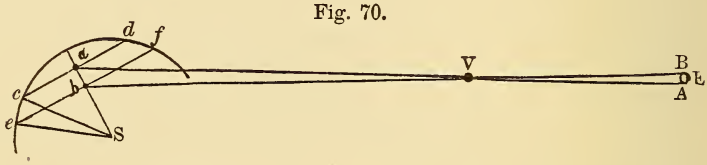
	Interval.
June 5th, 1518	
June 2d, 1526	8 years.
Dec. 7th, 1631	105½ "
Dec. 4th, 1639	8 "
June 5th, 1761	121½ "
June 3d, 1769	8 "
Dec. 8th, 1874	105½ "
Dec. 6th, 1882	8 "
June 7th, 2004	121½ "

280. *Parallax of the sun by a transit of Venus.*—The planet Venus is so near the earth, that its transit across the sun's disk is peculiarly favorable for obtaining the sun's parallax. Let E (Fig. 70) be the earth, V Venus, and *fde* the disk of the sun. Suppose observers stationed at A and B, the extremities of that diameter which is perpendicular to the orbit of Venus. Each one sees the planet describe a chord across the sun's disk from east to west. A observes it to come on at *c*, and leave at *d*; while to the view of B, it comes on at *e*, and leaves at *f*. And when it appears at *a* to the former, it is seen at *b* by the latter. It is the distance between the two projections at *a* and *b* which is to be determined.

281. *The length of* ab *in miles.*—Since the periodic times of the earth and Venus are known, the ratio of the distances of E and V from the sun is also known, by Kepler's third law. Hence, by subtraction, the ratio of the lengths of the triangles

VA and Va is known. These triangles may be regarded as isosceles; therefore, as they have equal angles at V, they are similar. Hence, VA : Va : : AB : ab. Thus, from the known ratio of VA to Va, and the length of AB, we have the length of ab in miles.

Fig. 70.



282. *The length of* ab *in seconds.*—We next wish to obtain the angular length of ab. The observers carefully mark the moment of entering on the disk, and the moment of leaving it. Thus, the length of time occupied by the transit, as seen by each observer, is carefully obtained. But since the angular motion per hour, both of the planet and the sun, is known, the time of crossing the disk can be changed into an arc; and we thus have the number of seconds of a degree in the chord cd, and also the number in ef, and, therefore, in their halves, ca and eb. But the number of seconds in the sun's semi-diameter, cS or eS, is known. Hence, in the right-angled triangles cSa, eSb, we readily find the seconds in Sa and Sb, the difference between which is the length of ab in seconds. Thus, we find what angle is subtended by a line of given length, when placed at the sun, and viewed from the earth; or, which is the same thing, placed at the earth, and viewed from the sun. Therefore, we know what angle at the sun is subtended by the radius of the earth; and that is the sun's horizontal parallax.

283. *External and internal contacts.*—At inferior conjunction, the planet Venus subtends an angle of more than 1′, and therefore, in the transit, appears like a small black circle, whose diameter is $\frac{1}{30}$ of the sun's diameter. To observe the beginning and end of a transit, the instant of external contact must first be noted, and afterward, when the planet has come wholly upon the disk, the time of internal contact also. The mean of these is the time at which the center crosses the edge of the

disk. The duration of the transit is the interval between the moments at which the center of the planet enters and leaves the disk.

284. *Situation of the observers.*—The observers can not probably be at points diametrically opposite, nor can they remain stationary during the transit, on account of diurnal motion; therefore, allowance must be made for these circumstances. In order that several independent results may be obtained, many stations are chosen, at the greatest possible distance from each other. In the observations on the transit of 1769, one of a large number of stations was in Lapland, and another on one of the Sandwich Islands. The result arrived at was, that the sun's horizontal parallax is 8''.5776, which, however, is now considered to be too small. (See Preface.)

MARS.

285. *Tabular statements.*—Mean distance from the sun, 140,760,000 miles; periodic time, 2 years; diameter, 4,211 miles; diurnal rotation, 24.62 hours; specific gravity, 4.17.

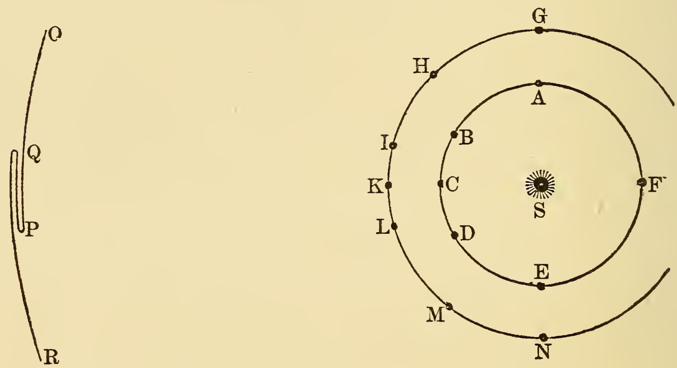
286. *Situation of Mars in the solar system.*—This is the most remote planet of the first group described in Art. 256—namely, Mercury, Venus, Earth, Mars. It is also the nearest to the earth of those planets which are called superior.

As Mars revolves in an orbit outside of the earth's, it can come into *opposition* to the sun, as well as into conjunction with it, appearing at every degree of elongation from 0° to 180°.

287. *Apparent motions.*—The real motion of Mars is from west to east; and during most of the year, its apparent motion is in the same direction, sometimes accelerated, and sometimes retarded, by the earth's motion. Near opposition, however, when the earth overtakes and passes by Mars, its motion appears retrograde. Thus, let the earth make one revolution from F to F again (Fig. 71), while Mars describes nearly a half revolution from G to N. When the earth is at F, Mars ap-

pears in the direction FG; when at A, Mars at H, appears in the sky at O; when the earth is at B, Mars at I, appears at P. Thus far, the motion has been in advance, though becoming retarded near P. But as the earth passes from B, through C, to D, Mars, passing over the shorter arc IKL, appears to retrograde from P to Q; after which it again advances, appearing at R when the earth is at E, and in the direction FN when the earth is at F.

Fig. 71.



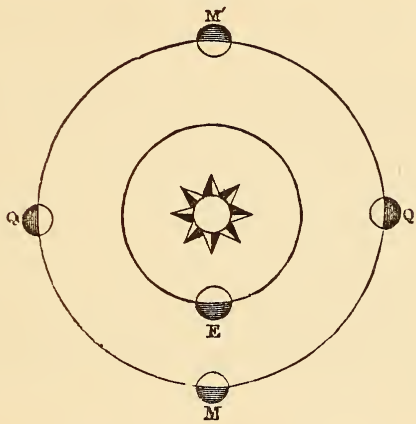
For the same reason, all the superior planets have a retrograde motion at the time of opposition.

288. *Phases, and changes of apparent size.*—At opposition, M (Fig. 72), and at conjunction, M′, it is obvious that Mars appears full, since we look in the same direction in which the sun shines upon it. In other positions, the angle between the sun's rays and our visual line is acute, and the phase is gibbous (Art. 170). The planet is so near us, that the phase differs perceptibly from the full, when about half-way from conjunction to opposition, as at Q, Q′.

At opposition, Mars is nearer to us than at conjunction by the diameter of the earth's orbit. This makes its mean distance at opposition 48,000,000 miles, and at conjunction, 233,000,000. But on account of the elliptical form of both orbits, the least distance is 34,000,000 miles, and the greatest, 247,000,000 miles.

289. *Orbit and equator of Mars.*—The orbit of Mars is inclined to the ecliptic nearly 2°, and has an eccentricity equal to $\frac{1}{11}$.

Fig. 72.



In its diurnal rotation, it considerably resembles the earth, having about the same length of day, and its equator being inclined nearly 29° to its orbit. Hence, the seasons vary somewhat more than those on the earth.

290. *Appearance of disk.*—Mars is remarkable among the planets for its redness. The telescope reveals some permanent inequalities of surface, by which its diurnal rotation has been determined more satisfactorily than in the cases of Mercury and Venus. And there are other appearances, which change as the relation of the equator to the sun changes. The polar regions, when turned away from the sun, exhibit a whiteness, which is supposed to be the effect of ice and snow; and this whiteness disappears gradually, when the pole is turned again toward the sun.

290a. *Satellites.*—Mars has two satellites, discovered August 11–17, 1877, by Prof. Asaph Hall, of Washington. They are remarkable for their small size, and for the proximity of the inner one to the planet; being but 4,000 miles from its surface, and consequently revolving in the brief time of 7 hours, 38 minutes.

CHAPTER XVI.

THE PLANETOIDS.—JUPITER.—SATURN.—URANUS.—NEPTUNE.

291. *The space between the four small planets and the four large ones.*—The large interval between Mars and Jupiter, which seemed to break the continuity of the series of planets, was noticed by Kepler. About the close of the last century, Bode, of Berlin, showed that a series of numbers, following a certain law, would express pretty accurately the planetary distances from the sun, if only the vacancy between Mars and Jupiter were supplied. This led to a special search for new planets, which was presently rewarded by the discovery of several small bodies, which have been called asteroids, planetoids, or minor planets.

THE PLANETOIDS.

292. *Their number, and the time of their discovery.*—Four of these bodies were discovered within the first seven years of the present century—namely: Ceres, Pallas, Juno, and Vesta. Since 1845, others have been found nearly every year, till their number at the present time (1884) is over two hundred. The whole number of planetoids may be regarded as indefinitely great.

293. *Characteristics.*—They are distinguished from the eight planets in the following particulars:

1. *By their diminutive size.*—They are invisible to the naked eye, and by the telescope can not be distinguished from faint fixed stars, except by their motion. They are generally too small to show a sensible disk, and hence can not be measured with any certainty. The largest of them is believed to be only about 200 miles in diameter. And it is estimated by the slight disturbing influence which they exert, that their entire mass is equal only to a small fraction of the earth.

2. *By the large eccentricity and obliquity of their orbits.*—The eccentricity of most of them is much greater than that of any of the eight planets.

The obliquity of the orbit of Hebe is 14°, and that of Pallas is 34°, which is the greatest yet discovered.

3. *By their being clustered in a ring.*—The orbits vary considerably in size, and therefore the periodic times are various But as they are generally quite eccentric, nearly every planetoid is nearer the sun at perihelion, than the others at aphelion. The orbits are therefore all linked together, and pass through each other. Thus, the planetoids are to be regarded as moving among each other about the sun, within the limits of a ring, whose breadth, in the direction of the radius vector, is more than 160,000,000 miles. Flora, which moves in the smallest orbit yet discovered, performs its revolution in $3\frac{1}{4}$ years; Hilda, the most remote, in 8 years. Their mean periodic time is $4\frac{1}{2}$ years; and their mean distance from the sun is 250,000,000 miles.

294. *Modes of designating them.*—Feminine mythological names have been applied to all the planetoids which have yet been discovered. But the more convenient method, and the one most used, is to express each planetoid by a number, showing its place in the order of discovery, this number being inclosed in a circle, which indicates a disk. Thus, Ceres is (1); Thetis, (17); Pandora, (55); etc. See Table V., at the end.

JUPITER.

295. *Tabular statements.*—Mean distance from the sun, 480,638,000 miles; periodic time, 12 years; diameter, 86,657 miles; diurnal rotation, 9.92 hours; specific gravity, 1.3.

296. *Jupiter's magnitude and place in the solar system.*—Jupiter is the nearest of the large planets outside of the planetoids, and its orbit is not far from 130,000,000 miles beyond the ring which includes them. On account of its great distance from the sun, compared with the earth's, Jupiter presents to us no visible change of phase, appearing always full. Its disk, as

presented to us, is almost the same as if we were at the sun The same is, of course, true of all the planets still more remote.

Jupiter greatly surpasses all the other planets in magnitude. In volume, it is about $1\frac{1}{2}$ times the sum of all the others, and in mass, more than $2\frac{1}{3}$ times their united mass.

297. *Its spheroidal form.*—Though the diameter of Jupiter is 11 times that of the earth, yet it rotates on its axis in less than 10 hours; so that the equatorial velocity is about 27 times as great as the earth's. This rapidity of rotation produces a sensible oblateness of the planet. Its ellipticity is $\frac{1}{17}$; and so considerable a deviation from the spherical form is perceptible to the eye without measurement.

298. *The belts of Jupiter.*—This name is given to bands or stripes of darker shade than the rest of the disk, stretching across it in the direction of its rotation (Fig. 4, Fr.) They vary from time to time in number and in breadth, often covering a large part of the surface. A belt usually appears of uniform breadth entirely across, but not always; its edge is occasionally broken, and sometimes it is much wider on one part of the disk than on the other, the change of breadth being commonly quite abrupt, and thereby revealing the rotation of the planet. There are, ordinarily, two conspicuous belts, lying near the equator, one north, and the other south of it.

299. *Supposed cause of the belts.*—The belts are considered as affording proof that Jupiter is surrounded by an atmosphere, in which clouds are floating. As a consequence of the exceedingly rapid rotation of the planet, there would be very powerful currents, analogous to the trade-winds of the earth; and the clouds would be thrown into the form and arrangement of zones parallel to the equator. The clouds would reflect the sun's light to us more strongly than the atmosphere; and the dark belts, therefore, are the unclouded portions, through which we look on the body of the planet.

300. *Orbit and equator of Jupiter.*—The orbit of Jupiter

is nearly coincident with the plane of the ecliptic, its inclination being only 1° 19′. Its eccentricity is $\frac{1}{20}$, which is three times as great as that of the earth's orbit.

The equator of Jupiter is inclined a little more than 3° to its orbit. There is, therefore, no perceptible change of seasons on that planet.

301. *Satellites of Jupiter.*—These are four in number, revolving in orbits very nearly circular, and in planes which make small angles, both with the orbit and the ecliptic. They are called the *first*, *second*, *third*, and *fourth*, reckoning outward from the planet.

302. *Their revolutions.*—On account of the position of the orbits, we see the satellites passing back and forth across the place of Jupiter, nearly in straight lines (Fig. 4, Fr.) From their greatest elongation west of Jupiter, they advance to the greatest elongation on the east, passing *behind* the planet on their way. Then, after remaining stationary a short time, they retrograde to the west side, passing between us and the planet. These movements prove that they revolve from west to east, as all the primary planets do. At the greatest elongation on the east side, they are, for a little while, stationary, because coming toward us; and on the west side also, because going from us.

303. *Their size, distance, and periods.*—The diameters of the first, third, and fourth satellites are greater than that of the earth's moon, but the diameter of the second is a few miles less. To us, they appear as stars of the 6th or 7th magnitude; but on account of the brightness of the primary, they can very rarely, if ever, be seen by the naked eye. If two or three satellites happen to appear very near together, they may possibly be seen by the naked eye, when they of course seem to be one. The first is further from Jupiter than the moon is from the earth, and the fourth nearly five times as far. Their periods of revolution are very short, compared with the moon's; for, on account of the strong attraction of Jupiter, great velocity is requisite to maintain them in their orbits.

Satellites.	Diameters.	Distances.	Sidereal Revolutions.
1	2,365	260,370	1 d. 18 h. 28 m.
2	2,123	414,360	3 " 13 " 15 "
3	3,471	660,900	7 " 3 " 43 "
4	2,966	1,162,400	16 " 16 " 32 "

304. *Their configurations.*—The relative positions of Jupiter and the four satellites, as seen from the earth, are incessantly varying. We most frequently see two or three on one side, and two or one on the other; rarely all on one side. Very often, one or two are invisible, being either behind Jupiter, or projected on it. Sometimes, three are thus concealed, and in very rare instances, all four.

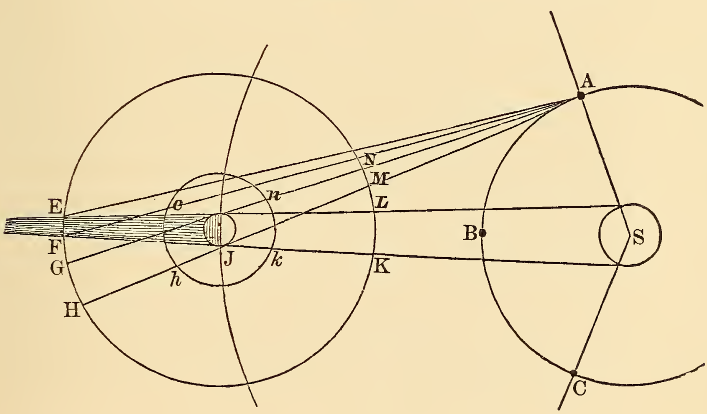
305. *Eclipses and occultations of Jupiter and its satellites.*—The great dimensions of Jupiter and its shadow, and the small inclinations between the ecliptic, Jupiter's orbit, and those of its satellites, cause very frequent eclipses and occultations. A satellite of Jupiter is eclipsed when it goes through the shadow of the planet; it suffers occultation when it is hidden from our view by passing behind the planet. The first, second, and third satellites pass through both eclipse and occultation at every revolution, and the fourth rarely escapes.

Besides these two classes of phenomena, there are two others, —namely, the eclipse of Jupiter, when its satellite casts a shadow upon it; and an occultation of Jupiter, when a satellite passes between it and the earth. The eclipse is a small black spot passing over the disk. The occultation is scarcely perceptible, because the planet and satellite are of about equal brightness. On a belt, the satellite may appear brighter and between two belts it may appear less bright than the primary.

306. *Order of eclipses and occultations.*—When Jupiter is *east* of opposition, the eclipse always precedes the occultation; when *west* of opposition, the occultation precedes the eclipse. For, let S (Fig. 73) be the sun; A, B, C, several positions of the earth; J, Jupiter; and EHK, the orbit of a satellite. The

bodies are supposed to revolve in the order of the letters. If the earth is at A, SA produced marks the place of opposition, and Jupiter is east of that place. The satellite enters the shadow at E, emerges at F, and then passes behind the planet at G, and reappears at H. In this case, the eclipse is past before the occultation begins. In the same manner, the eclipse of Jupiter begins when the satellite is at K, and ends when at L; and the occultation follows it, while the satellite moves from M to N. If the earth were at C, Jupiter would be west of opposition—that is, west of SC produced. And it is obvious that the satellite would go behind the planet before entering the shadow, and also would appear between us and the planet before casting a shadow on it.

Fig. 73.



The earth is not, in general, so situated that one phenomenon is closed before the next begins; and it is never true of the first satellite. The case is represented by the orbit *ehkn*. The eclipse begins at *e*, and the occultation ends at *h*; but the end of the eclipse and the beginning of the occultation are not seen. In the same manner, the eclipse of Jupiter begins at *k*, and the occultation ends at *n*. During a part of the intervening time, the shadow and the body of the satellite are both seen, projected at different places on the primary.

At the time of opposition, the earth being at B, the eclipse

of a satellite obviously occurs entirely within its occultation, and the occultation of Jupiter entirely within its eclipse.

It is found that there exists such a relation between the mean motions of the three first satellites, that they can never all be eclipsed at the same time.

307. *The velocity of light discovered by the eclipses of Jupiter's satellites.*—In 1675, it was discovered by Roemer that eclipses occurred earlier than the calculated time, when the earth is in that part of its orbit which is near to Jupiter, and later, when in the remote part. The eclipses of any one satellite are so frequent, that the *mean* interval between them is obtained with great accuracy; and by this mean interval, the times of future eclipses could be calculated. But it was perceived that while the earth moves from the remote side to the nearer side of its orbit, the *real* intervals are shorter than the mean, so that, at the nearest point, an eclipse occurs about 8m. 13½s. too soon. Again, as the earth goes to the side of its orbit furthest from Jupiter, the real intervals are all greater than the mean; and at the most distant point, an eclipse is later than the calculated time by 8m. 13½s. Roemer attributed this periodical error of time to the progress of light, and inferred that light requires 16m. 27s. to cross the earth's orbit. This makes the velocity of light near 187,000 miles per second; which seemed at first quite incredible, and was received with distrust. But its correctness was soon established by the discovery of the aberration of the stars, which gives about the same result (Art. 146).

SATURN.

308. *Tabular statements.*—Mean distance from the sun, 881,203,000 miles; periodic time, 29 years; diameter, 70,500 miles; diurnal rotation, 10.24 hours; specific gravity, 0.8.

309. *Saturn's disk.*—Saturn is the second planet in size; and being the second in order beyond the planetoids, is not too far from the earth to present a large disk. Its form is seen to be elliptical, and it is faintly striped with belts in the direction

of the major axis. Both these appearances are explained by the rapid rotation of the planet on its axis, as in the case of Jupiter. Its ellipticity is $\frac{1}{10}$.

310. *Saturn's rings.*—The distinguishing feature of this planet is the system of broad thin rings which surround it. They lie in a plane inclined about 28° to the ecliptic, and therefore generally present an elliptical appearance to the earth (Fig. 3, Fr.) The ring, as usually seen, consists of two rings, the inner of which is the widest. The inner edge is 19,000 miles from the surface of the planet; and the diameter from outside to outside is 168,000 miles. The line in which the plane of the ring intersects the plane of Saturn's orbit is called the line of the nodes.

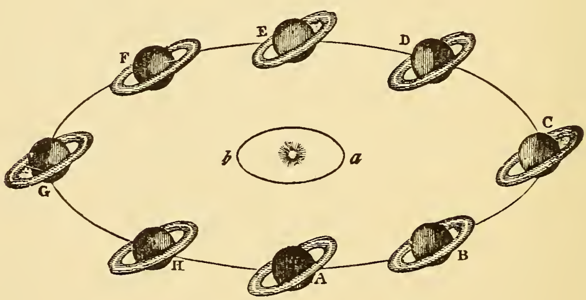
Within the double ring already described, there is a much fainter one, which can not be seen with ordinary telescopes. By careful observations, it is also perceived that there are several concentric divisions of the rings, which vary their number and position from time to time. These fainter divisions are invisible, except at the ends of the ellipse. The rings lie in one plane, and are exceedingly thin. The latest measurements make their thickness less than 40 miles. A circle of common writing paper, one foot in diameter, would be too thick to represent it correctly. But the thickness appears not to be uniform; for in the edge view, it often presents the aspect of a broken line, as though some parts were thick enough to be seen, and others not. There seems to be evidence that the rings consist either of liquid matter, or else of solid matter in a disintegrated condition.

311. *Rotation of the rings.*—Such rings of matter around Saturn could no more be sustained without rotation, than the moon could remain at its distance from the earth without revolving about it. They are found to rotate in their own plane within the short period of $10\frac{1}{2}$ hours, nearly the same as the period of the planet itself. The outer edge of the ring must, therefore, have a velocity of 14 or 15 miles per second.

312. *The plane of the rings always parallel to itself.*—

During the revolution of Saturn around the sun, occupying about 29 years, the rings maintain everywhere the same position in relation to the plane of Saturn's orbit as represented in Fig. 74, in which *ab* is the earth's, and ACEG Saturn's orbit, seen obliquely. While the planet passes through the half revolution ACE, the north side of the rings is seen by an observer on the earth as an ellipse, more or less eccentric; but during the other half, EGA, the south side is in view. Each of these periods occupies near 15 years. When Saturn is near A and E, the line of nodes passes across the earth's orbit, and the edge of the rings is therefore directed toward the sun and earth; and at those times it fills too small an angle to be seen, except by the best instruments.

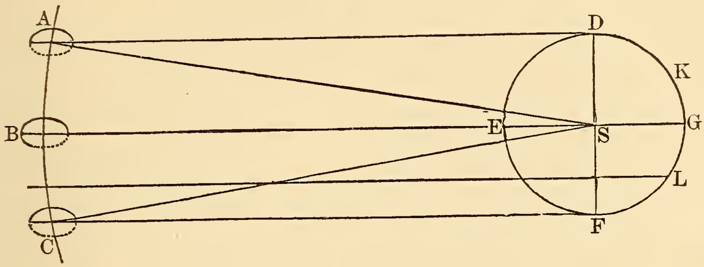
Fig. 74.



313. *Passage of the plane of the rings across the earth's orbit.*—The motion of Saturn is so slow, that it requires almost a year for the plane of its rings to pass by the whole diameter of the earth's orbit. Let DF (Fig. 75) be the earth's orbit, and AC a portion of Saturn's. Suppose these orbits to lie in the plane of the paper, and the plane of the rings to be inclined about 28° to the paper, making the common section of the two planes in the lines AD, BG, etc. Saturn is 9.54 times as far from the sun as the earth is. Therefore, SA : SD : : 9.54 : 1 : : rad. : sin SAD; ∴ SAD, or its equal, ASB = 6° 1′; ∴ ASC = 12° 2′. Knowing Saturn's periodic time, we readily find that it will describe 12° 2′ in 359½ days, near six days less than a year. Hence, while Saturn passes from A to C, the earth will pass very nearly around its orbit, DEFG. But the earth

may be at any point of its orbit when the planet reaches A. The disappearances of the rings will vary according to the positions of the earth.

Fig. 75.



314. *Circumstances of the disappearances.*—There are three ways in which the rings may fail to be visible during the period in which the line of their nodes is crossing the earth's orbit.

1. The ring may present its edge exactly to the *earth*, when, in common telescopes, it subtends too small an angle to be seen.
2. It may present its edge exactly to the *sun*, so that neither side of the ring is enlightened.
3. Its plane may be directed *between* the earth and sun, when the dark side is toward us.

The disappearance by either of the two first causes may be considered as only momentary; for the line of nodes passes the breadth of the sun in less than 2 days, and of the earth in about 20 minutes. But the third cause may conceal the ring from our view for weeks or months. This prolonged disappearance may occur either once or twice, or possibly not at all, while the line of nodes is passing the breadth of the earth's orbit.

315. *One disappearance.*—If the earth is at F when the planet reaches A, then the earth will go from F nearly to D, while the nodal line advances from AD to BS, and the earth will pass the line between G and D, as at K. Up to that point, the luminous side is presented toward the earth; but from K to a point near D, the plane of the rings falls between the earth and sun, and the rings are invisible, and continue so

about two months. When the nodal line has passed the sun, the luminous side of the rings is again toward the earth; and before the earth completes the half orbit DEF, the nodal line will pass off at F.

316. *Two disappearances.*—If the earth has advanced some distance on the quadrant FG—for example, to the middle L—when the nodal line touches D, then the earth passes the line between K and D, and the dark side is toward us. The line passes the sun when the earth is near the middle of DE, after which, the rings are seen. But before the nodal line reaches CF, the earth will overtake it, and be on the dark side again. Between F and L, the earth once more crosses the line, and the rings present to us their bright side. In this case, the rings disappear twice during the nodal year.

These two periods of disappearance may be so prolonged as to unite in one of about eight months in length. This happens when the earth is two or three days past G, at the time when the nodal line touches D. Then, before reaching D, the earth passes to the dark side of the rings, and continues on that side till both the earth and the nodal line pass E together. As soon as that point has been passed, the line is again between the sun and earth, and continues so until it is recrossed by the earth on the quadrant FG.

317. *No disappearance.*—It is possible that no disappearance, which has continuance, should happen during the nodal year. Suppose the earth two or three days past E, when the line of nodes reaches D. Then, while the line moves from AD to BS, the earth will advance to G, all the time on the luminous side of the rings; the earth and sun will both be in the line BSG at once, the planet being in conjunction; and after the earth has passed G toward D, the bright side of the rings is in view, as before, and will continue so. Thus, there is only a momentary disappearance, and that, when the planet and rings are lost in the blaze of the sun's light.

In general, there are two periods of disappearance within the nodal year, arising from the third cause, each beginning and ending with a disappearance from the first or second cause.

318. *Phenomena of the rings at the planet.*—On that hemisphere of the planet to which the luminous side of the rings is presented, there is the appearance of splendid arches spanning the sky, having a breadth and elevation according to the latitude of the place. At latitude 30°, the breadth is about 18°, and the elevation of the lower edge on the meridian about 22°. Near the poles, however, it is below the horizon. The luminous side is presented to the northern hemisphere near 15 years, and then the same length of time to the southern hemisphere, in regular alternation.

A part of the rings is generally eclipsed by the shadow of the planet falling on it.

Also, during the 15 years in which the dark side of the rings is turned toward a hemisphere, its shadow is cast across a zone of it, which causes an eclipse of the sun. And at a given place, a total solar eclipse may continue from day to day, without interruption, for several years.

319. *Satellites of Saturn.*—Saturn is attended by eight satellites. Their periods of revolution vary from less than one day to 79 days. Their diameters vary from 500 to 2,900 miles; but on account of their immense distance from the earth, they are seen only with the best instruments. They are all external to the rings, at distances from the planet, varying from 122,000 to 2,338,000 miles. Their orbits are nearly in the plane of the rings, and make an angle of about 28° with the orbit of the planet. Hence, they are not very liable to be eclipsed. The principal time for eclipses is that at which the rings disappear; for then the sun is nearly in the plane of their orbits, as well as of the rings.

URANUS.

320. *Tabular statements.*—Mean distance from the sun, 1,772,088,000 miles; periodic time, 84 years; diameter, 31,700 miles; specific gravity, 1.3.

321. *Discovery, and place in the system.*—Uranus was unknown to the ancient astronomers; and to them, therefore,

Saturn's orbit was the boundary of the solar system. Uranus was discovered by Sir William Herschel, in 1781, and has made but little more than one revolution since that time. It was, however, repeatedly seen by earlier astronomers, and recorded in their catalogues as a fixed star. By this discovery, the diameter of the known solar system was doubled.

Uranus is the third of the four great planets in order of distance, but it is least in diameter. Its distance from us is so immense that it appears only as a faint star, and presents no inequalities by which its diurnal motion can be discovered. Its orbit is very nearly circular, and is inclined less than a degree to the ecliptic.

322. *The satellites of Uranus.*—Sir William Herschel announced the discovery of *six* satellites belonging to Uranus. But only *four* have been identified by later astronomers. The remarkable facts relating to these satellites are, that their orbits are nearly at right angles to the plane of the ecliptic, and that *in* the orbits, the motions of the satellites are *retrograde*—that is, from east to west. Their periods of revolution vary from 2½ days to 13½ days, and their distances from 123,000 to 376,000 miles.

NEPTUNE.

323. *Tabular statements.*—Mean distance from the sun, 2,777,948,000 miles; periodic time, 165 years; diameter, 34,500 miles; specific gravity, 1.1.

324. *Discovery.*—Neptune was discovered in 1846. The circumstances which led to the discovery were briefly as follows. After the orbit of Uranus had been carefully computed, and corrections made for the disturbing influence of Jupiter and Saturn, the planet was found to depart from the calculated path in a manner not to be accounted for, except by supposing some other disturbing force. It was for some time suspected that there must be a planet superior to Uranus, whose attraction caused the change of its orbit. At length, two mathematicians, Le Verrier, of France, and Adams, of England,

each without any knowledge of what the other was attempting, engaged in the arduous labor of calculating what must be the elements of a planet which should produce the given disturbance of the motions of Uranus. They reached results which agreed remarkably with each other. Le Verrier communicated to Galle, of the Berlin observatory, the place in the sky in which the disturbing body should be situated; and in the evening of the same day, Galle found it within a degree of the predicted longitude.

The planet thus discovered explains fully the disturbances in the motions of Uranus.

It soon appeared that Neptune had repeatedly been entered in catalogues as a fixed star. The earliest of these records, in 1795, afforded material aid at once in determining its mean distance and its periodic time.

Neptune is attended by one satellite, which was also discovered in 1846. It is nearly as far from the primary as the moon is from the earth, and revolves in 5d. 21h.

CHAPTER XVII.

ELEMENTS OF A PLANETARY ORBIT.—QUANTITY OF MATTER IN THE SUN AND PLANETS.—PLANETARY PERTURBATIONS.—RELATIONS OF PLANETARY MOTIONS.

325. *Elements of an orbit.*—These are the quantities which must be known, in order to calculate the place of a planet at a given time. They are *seven* in number.

1. The periodic time.
2. The mean distance from the sun, or the semi-major axis of the orbit.
3. The longitude of the ascending node.
4. The inclination of the plane of the orbit to that of the ecliptic.
5. The eccentricity of the orbit.
6. The longitude of the perihelion.

7. The place of the planet in its orbit at a given epoch.

Two of these, 3d and 4th, determine the position of the plane in which the orbit lies; the second fixes the size of the orbit; the 5th, its form; the 6th, the relation of the form to the plane of the ecliptic; the 1st and 7th, the circumstances of the planet's motion in the orbit.

The orbit of a planet can not be determined by the same method as the moon's orbit is (Chap. X.), or the sun's apparent orbit (Chap. IV.), because it is not the earth, but the sun, which occupies the center of the planetary revolutions.

326. *Geocentric and heliocentric place of a planet.*—The point in the celestial sphere which a planet occupies, as seen from the *earth*, is called its *geocentric* place; its place as seen from the sun is called its *heliocentric* place. It has already been noticed that the planets, as seen from the earth, have a retrograde motion during a part of every synodical revolution. This is the effect of the observer's position and motion, and would not exist if he were stationed at the sun. The place of a planet, as seen from the earth and the sun, can never agree, except when the sun and earth are on the same side of the planet, and in the same straight line with it. But after the relations of the earth to the planet and to the sun are obtained, there is no difficulty in calculating the heliocentric place of the planet.

327. *First element—the periodic time.*—This is found by observing the time that intervenes between the two successive returns of the planet to the same node.

It may be known when a planet is at a node, because then its latitude is nothing. If, from a series of observations on the right ascension and declination of a planet, the latitudes are computed, and one of them is *zero*, then the exact time of passing the node is obtained. But if, as is usually the case, the two least latitudes are, one north, and the other south, the time of passing the node between them is readily found by a proportion. Similar observations are made when the planet again arrives at the same node, and thus the periodic time becomes known.

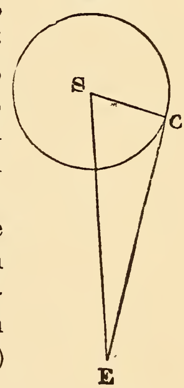
It is discovered that a minute correction of the periodic time,

thus derived, must be applied for the retrograde motion of the node. The periodic time of a planet may also be derived from the observed length of its synodic revolution—that is, the interval between two successive oppositions, or two conjunctions of the same kind. The computation is similar to that employed in finding the sidereal period of the moon from its synodical period (Art. 158).

In both the above methods, great advantage, in point of accuracy, is gained, if two very distant epochs can be brought into comparison, such as two distant passages of the node, or of opposition. For example, a transit of Mercury occurs at inferior conjunction. Divide the interval between two observed transits, several years apart, by the number of synodical revolutions of Mercury which intervene, and its mean synodical period is very accurately obtained.

328. *Second element—the distance from the sun.*—The distance of an inferior planet from the sun is found as follows Let S (Fig. 76) be the sun, E the earth, and C the planet. Measure the greatest elongation, SEC; then, in the right-angled triangle, rad : sin SEC : : SE : SC. If the orbit is elliptical, the value of SC, as obtained at different times, will be different; and a great number of such observations should be made, in order to obtain the *mean* distance.

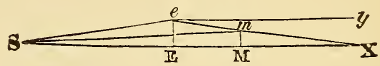
Fig. 76.



The distance of a superior planet may be found by observations on its retrograde motion at the time of opposition. For, the more distant the planet, the less will the earth's motion throw it, apparently, backward. Let S (Fig. 77) be the sun, E the earth, and M a superior planet. Let E pass over E*e* in a short time, as one day, and let M pass over M*m* in the same time. As the periodic times of E and M are supposed to be known, the angles ES*e* and MS*m* are known, and, therefore, their difference, *e*S*m*. Join *em*, and produce it to X in SM produced. Draw *ey* parallel to SX; the angle X*ey* is the retrogradation during the day in which the planets describe the arcs E*e* and M*m*, and is known by observation. But SX*e* = X*ey*; and, therefore, in the triangle

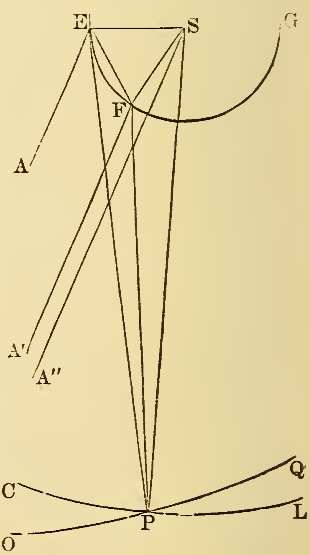
*e*SX, the third angle, X*e*S, is known. Hence, in the triangle S*em* we have all the angles, and the side S*e*, by which S*m* is easily computed. This process may be repeated at every opposition, and thus the *mean* distance is ultimately obtained.

Fig. 77.



329. *Third element—longitude of the node.*—Let S (Fig. 78) be the sun, EFG the earth's orbit, OPQ the orbit of a planet, CL an arc in the plane of the ecliptic, intersecting the orbit in P. SP is, therefore, the line of the nodes. And let EA, FA′, and SA″ be parallel lines, directed toward the vernal equinox. When the earth is at E, suppose the planet is at the node P; then E, P, and S are all in the plane of the ecliptic, and AEP is the longitude of P, and AES that of the sun. These longitudes being obtained, their difference is SEP, which is, therefore, known. After the planet has performed a revolution to the same node again, suppose the earth to be at F; then we find, as before, its longitude, A′FP, that of the sun, A′FS, and their difference, SFP. As the times are known in which the earth is at E and at F, we know SE, SF, and the angle ESF, and can compute EF, and the angles SEF and SFE. From the several angles at E and F, thus obtained, we derive PEF and PFE; and these, with the side EF, give us the side FP. Then, in the triangle SFP, from SF, FP, and SFP, compute the angle FSP. From this, subtract A″SF (the supplement of A′FS), and there remains A″SP, the heliocentric longitude of the node.

Fig. 78.



It is by processes of this kind that the slow retrograde mo-

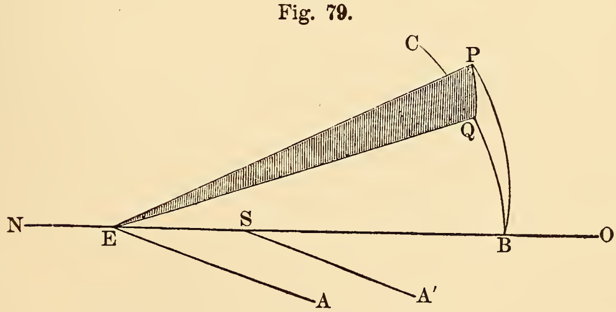
tion of the nodes is discovered (Art. 327). It amounts to only a few minutes in a century.

330. *Fourth element—inclination of the orbit to the ecliptic.*—Select the time of observation, when the sun's longitude, obtained from the tables, is the same as the heliocentric longitude of the node; and find for that time the geocentric longitude and latitude of the planet. Let E (Fig. 79) be the earth S the sun, P the planet, NO the line of the nodes coinciding with ES; and let EA and SA′ be the direction of the vernal equinox. Join EP, and, with it as a radius, describe the surface of a sphere, cutting the plane of the ecliptic in the arc BC. From P draw the arc PQ, perpendicular to BC. AEO is the longitude of the sun, and A′SO, its equal, is the heliocentric longitude of the node O. AEQ is the geocentric longitude of the planet. In the spherical triangle BPQ, right-angled at Q, PQ measures the given latitude, BQ measures the difference between AEQ and AES, and PBQ is the inclination to be found. Then, rad × sin BQ = tan PQ × cot PBQ;

$$\therefore \cot PBQ = \frac{\text{rad} \,.\, \sin BQ}{\tan PQ};$$

and the inclination of the orbit to the ecliptic becomes known.

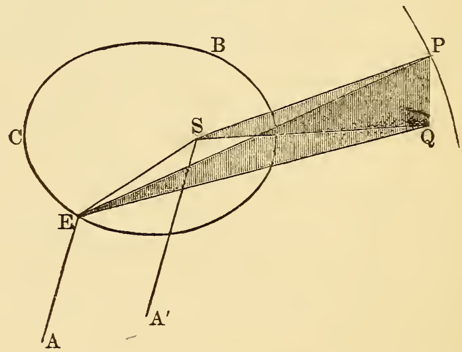
Fig. 79.



331. *To find the heliocentric longitude and latitude of a planet.*—Let S (Fig. 80) be the sun, E the earth, EBC its orbit, P the planet, EA, SA′ the direction of the vernal equinox. Let PQ be drawn perpendicular to the plane of the ecliptic. AEQ is the geocentric longitude of the planet, A′SQ its helio-

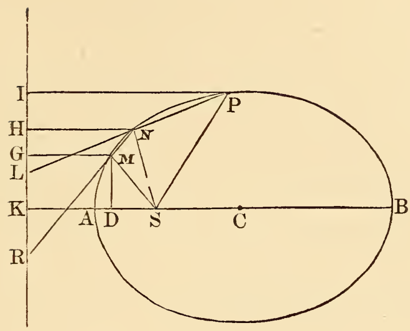
centric longitude. Also, PEQ is its geocentric latitude, and PSQ its heliocentric latitude. SEP, the elongation of the planet from the sun, is known from observation; SE, the radius vector of the earth's orbit, and SP, that of the planet's orbit, are also known. Therefore, PE may be computed. Knowing PE, and PEQ in the right-angled triangle, we can compute EQ. Then, in the triangle QES, EQ, ES, and the angle QES (= AES − AEQ) being known, QSE and QS are found. From QSE, subtract ESA′ (the supplement of AES), and A′SQ is obtained, which is the heliocentric longitude of P. Again, in the right-angled triangle PSQ, having SQ and SP, we find the angle PSQ, the heliocentric latitude.

Fig. 80.



332. *Fifth and sixth elements—eccentricity of the orbit, and longitude of the perihelion.*—A focus and three points in the curve of a conic section being given, its directrix can be determined, and the curve drawn. (Coffin's Con. Sec., Prop. II.) Thus, let SM, SN, and SP (Fig. 81) be three radii vectores of an orbit, determined in length and position by the processes already described. If MN and NP are joined, the triangles MNS and NPS are known in all respects. Then, if MN be produced, so that NR : MR : : NS : MS, R is a point of the directrix. Another point, L, is fixed in a similar manner by producing PN. The directrix being thus determined, draw perpendiculars to it from S, M, N, and P. The axis of the orbit is on KS produced. The ratio, SM : MG, is constant for every point of the curve. (Cof. Con. Sec., Pr. II.)

Fig. 81.



The distance SK of the focus from the directrix is found thus. Draw MD perpendicular to SK. LNS, the external angle of the triangle NSP being known, subtract MNS from it, and we have LNR, and the including sides LN, NR, to find the angle R. This, with the side MR, in the right-angled triangle MGR, gives us GM, and the angle GMR. Then, $180° - (GMR + RMS) = MSD$, from which, and MS, we compute DS; and $GM + DS = SK$.

To find the perihelion, divide SK, so that SA : AK : : SM : MG; A is the perihelion.

To find the aphelion, produce KS to B, so as to make SB : BK : : SM : MG; B is the aphelion.

Bisect AB in C; then SC divided by AC is the eccentricity of the orbit.

The longitude of the perihelion is known from the angle MSA, already obtained; for the longitude of SM is given at the outset.

333. *Masses of bodies compared by the orbits described about them.*—The mass of a body, whether the sun or a planet, can be compared with that of another, by means of the distance and period of a planet or satellite revolving about each. It has been proved (Chap. VIII.) that gravity varies directly as the mass, and inversely as the square of the distance; that is, $G \propto \frac{M}{D^2}$. It was shown, also (Chap. VI.), that the centripetal

force or gravity varies directly as the distance, and inversely as the square of the periodic time; that is, $G \propto \frac{D}{P^2}$. Therefore, $\frac{M}{D^2} \propto \frac{D}{P^2}$; $\therefore M \propto \frac{D^3}{P^2}$; or, the united mass of the central and the revolving body varies directly as the cube of their distance apart, and inversely as the square of the periodic time. Thus, to compare the mass of the sun, about which the earth revolves, and the mass of the earth, about which the moon revolves, we have

$$\frac{(238{,}820)^3}{(27.32)^2} : \frac{(92{,}381{,}000)^3}{(365.256)^2} :: 1 : 324{,}000, \text{ nearly.}$$

Therefore, the mass of the sun is about 324,000 times that of both earth and moon, or 327,000 times that of the earth alone.

334. *Examples.*—

1. Were the earth's mass equal to the sun's, in what time would the moon, at its present distance, revolve about it?

Letting x stand for the time required, we have $1 : 327{,}000 :: \frac{1^3}{(27.32)^2} : \frac{1^3}{x^2}$. *Ans.* 1h. 8m. 48s.

2. How much must the mass of the earth be increased, in order that the moon may revolve about it in the same time as it now does, when removed to three times its present distance? *Ans.* It must be 27 times as great.

3. The distance of Jupiter from the sun is 481,000,000 miles, and its periodic time is 4332.554 days. The fourth satellite is 1,162,000 miles from the primary, and revolves in 16d. 16h. 32m. Compare the mass of the sun with that of Jupiter. *Ans.* 1048 : 1.

4. The moon revolves in 27.32 days, at the distance of 238,820 miles from the earth: Jupiter's second satellite revolves in 3.552 days, at the distance of 414,000 miles. What are the relative masses of the earth and Jupiter?

Ans. 1 : 312.

335. *Masses of planets which have no satellites.*—The method described in the preceding article can be applied to

the sun, and all those planets which are attended by satellites. But Mercury and Venus, which have no satellites, must be compared in some other way. Each of these planets, by its attraction, sensibly disturbs the motion of the planet nearest to it; and the degree of this disturbance, the distance being known, is a measure of its quantity of matter. Thus, the masses of Venus and Mars can each be estimated, by observing the force which they exert on the earth when passing near it. The mass of Mercury has been determined by its disturbing power exerted on Encke's comet, as well as on the planet Venus.

336. *Densities of the planets.*—The masses of bodies vary as the products of their volumes and densities. Therefore their densities vary as the masses divided by the volumes. The densities, as given in Table IV, may be obtained in this way, and reduced to a scale, in which the earth's density is called 1. Or they may be reduced to a scale, in which the density of water is 1; in the last form, the numbers are called *specific gravities.* These also are given in Table IV

337. *Perturbations of the planets.*—The solar system, as we have seen, consists of many bodies; and each one of them attracts every other one, and attracts it more, according as it is nearer and more massive. Hence, no planet can continue to pursue the same elliptic orbit about the sun, as if the sun and planet were the only bodies. Nor can any satellite describe its orbit undisturbed about the primary. The number and variety of these disturbing forces exerted within the system are very great. But many of them are so minute as to be insensible. As was shown in Chapter X. respecting the moon, so in regard to every planet and satellite, the disturbing influences are of various kinds, some tending to alter the plane of the orbit, others to change its form, etc. These perturbations, like those of the moon, are classed into periodical and secular.

338. *Retrogradation of nodes.*—If the orbit of a planet is oblique to that of another, one component of the disturbing force tends to move the nodes of the two orbits backward, as

shown in Art. 192. And every satellite, whose orbit is inclined to that of its primary, is acted on by the sun, in the same manner as the moon is: its nodes retrograde on the orbit of the primary. For the planets, this retrograde motion is excessively slow, generally amounting to only a few minutes in a century.

339. *Change of inclination.*—Another disturbance of the orbit of a planet has respect to its inclination to the orbit of another. There are small periodical oscillations in the inclination, at every revolution, which nearly compensate each other, like those of the moon's orbit (Art. 193). But the compensation not being exact, there is a minute change, which remains unbalanced, and accumulates for many centuries, when the change is reversed and accumulates in the opposite direction. These secular oscillations, however, are all within narrow limits. Thus, the ecliptic, though generally spoken of as a fixed plane, is not truly so, but is subject to a minute change of a few seconds in a century. It is proved that the whole variation can never amount to 3°, and that within that range it will occupy many thousands of years in making a single secular oscillation.

340. *Advance of apsides.*—All planets within the orbit of a given planet, conspire, on the whole, to increase its gravity toward the sun; while the general effect of those outside of the same orbit is to diminish it. It was shown (Art. 183) that the sun, being outside of the moon's orbit about the earth, sometimes increases and sometimes diminishes the moon's tendency to the earth, but on the whole diminishes it. The same thing is true of every planet outside of the orbit of another. One consequence of the change in the attraction is, to cause the line of apsides to advance and to retrograde alternately; but the resultant of the whole action is an *advance.* The earth's apsides advance $11\frac{1}{2}''$ in a year (Art. 147). So the line of apsides of most of the planetary orbits has a slow motion from west to east.

341. *Change of eccentricity.*—A planet tends to increase the eccentricity of an orbit within its own, when the two

planets are in its line of apsides at conjunction and opposition, and to diminish it when the line from the sun to the outer planet makes a right angle with the line of apsides; analogous to the action of the sun on the moon's orbit (Art. 187). These disturbances are very minute, but they will not balance each other during a synodical revolution ; and therefore there is a small secular change in the eccentricity of the orbits. For example, the eccentricity of the earth's orbit has been diminishing, and for many thousands of years to come it will continue to diminish, at the rate of 0.00004 per century. The orbit, however, will never reach the exact form of a circle, but after arriving to a minimum of eccentricity, it will begin to return to a more eccentric form, and thus will oscillate about a mean value perpetually. And the range of its eccentricity is so limited, that the ellipse, if correctly represented, can never differ visibly from a circle.

It is this slow change in the earth's orbit which causes the secular inequality of the moon's motion (Art. 195).

342. *Change in the length of the major axis.*—There are also minute periodic changes in the length of the major axis of an orbit; that is, in the mean distance of a planet from the sun. But both calculation and observation establish the fact, that there is no *secular* inequality, because the periodical changes exactly compensate each other. And, if the mean *distance* of each planet from the sun has permanently the same value, then, according to Kepler's third law, the *periodic time* is also constant.

343. *Long periods.*—There are in the solar system several cases of inequality, accumulating for centuries, which nevertheless have the character of *periodical* rather than *secular* inequalities, and depend on the fact that the periodic times of two planets almost exactly measure a certain length of time.

For example, the earth makes 8 revolutions in very nearly the same time in which Venus makes 13. Hence, every fifth conjunction occurs within $1\frac{1}{2}°$ of the same points of their respective orbits. At the end of this period of 8 years, there is a minute perturbation, which remains uncompensated, and

which is about doubled at the end of 16 years, and tripled at the end of 24 years, and so on. This disturbance is very small, never amounting to more than a few seconds; but it requires a period of 240 years in order to pass through all its changes.

The long inequality of Jupiter and Saturn is a more remarkable case. Jupiter makes 5 revolutions, and Saturn 2, in nearly the same time. An unbalanced disturbance, which appears at the end of this time, goes on accumulating. During the 17th century, Saturn was constantly retarded, and Jupiter accelerated. But in the 18th, this was reversed, and Saturn is now accelerated, and Jupiter retarded. This will continue still longer; and the whole period required for this inequality is more than 900 years. The deviation, at its maximum, is 49′ for Saturn and 21′ for Jupiter.

344. *Degree of change in the several elements.*—Of the several elements named at the beginning of this chapter, we see, from what precedes, that the following classification may be made.

1. The 1st and 2d have no secular inequality whatever. Their value remains constant from age to age. The permanency of these two elements secures a constant length of the year, and a constant amount of heat from the sun on each planet.

2. The 3d and 6th elements have small periodical oscillations, but their secular change is in one direction; the nodes perpetually retrograde, the apsides perpetually advance. But the continual change in the same direction in these two elements has no tendency to derange the condition of things on a planet. As to the well-being of the occupants of a planet, it is of no consequence how the major axis of its orbit is situated, if only the form of the ellipse is preserved. It is also immaterial in what direction the line of its nodes may happen to lie.

3. The 4th and 5th elements have both periodical and secular inequalities; but they range within very narrow limits. The smallness of these changes insures all the planets against any considerable change from year to year, in respect to the extremes of heat and cold, and in respect to the seasons.

345. *Stability of the system.*—Several of the secular inequalities, before their true character had been demonstrated, excited great interest among astronomers, because they seemed to indicate the ultimate derangement of the order and stability of the system. If the eccentricity of the earth's orbit should continue to change in the same direction perpetually, the earth would at length, though perhaps not in millions of years, become unfit to be the habitation of man, because of the terrible extremes of heat and cold at perihelion and aphelion. So, if the inclination of equator and ecliptic should continue its change perpetually in the same direction as at present, the seasons would by and by disappear, and afterward run to an extreme which would produce desolation over the whole surface of the earth. And if the secular inequality of the moon's period were always to go on as it has done for centuries past, the moon would at length be precipitated on the earth.

But La Grange, La Place, and others have demonstrated that all the perturbations have their limits, and, after increasing with extreme slowness for many ages, must again decrease in like manner; and, furthermore, that the entire series of changes lies within so narrow bounds, that no disastrous consequences can ensue.

346. *Manner in which the stability is secured.*—The stability of the system is secured by the fulfilment of certain essential conditions in the arrangement of its parts.

1. The great mass of matter constituting the solar system is in the central body, the sun being 700 times as great as all the other bodies united. Hence, all movements are principally controlled by the sun.

2. The planets, and especially the large ones, are at great distances from each other; and thus the sun's influence over each is but little modified by their mutual attractions.

3. The orbits, especially of the largest planets, have but slight eccentricity, and, therefore, always maintain their great distances from each other.

4. The mutual inclinations of the orbits are small. Hence, there are no large forces operating to change the position of orbits, and thus disturb the seasons.

347. *Relations of the planetary motions.*—The planets are so adjusted to each other, in respect to their velocities, distances from the sun, periodic times, and gravity toward the sun, that if any one of these relations between two planets is known, all the others become known also.

Let r = mean distance; t = periodic time; v = velocity g = gravity. Also, let s (slowness) $= \frac{1}{v}$, the reciprocal of velocity; and l (lightness) $= \frac{1}{g}$, the reciprocal of gravity.

$$v = \frac{2\pi r}{t} \propto \frac{r}{t} \text{ (Art. 92)};$$

$$\therefore v^2 \propto \frac{r^2}{t^2}. \text{ But by Kepler's third law,}$$

$$t^2 \propto r^3; \therefore v^2 \propto \frac{r^2}{r^3}, \text{ or } v^2 \propto \frac{1}{r}.$$

As $s = \frac{1}{v}$, $\therefore v = \frac{1}{s}$, and $v^2 = \frac{1}{s^2}$; $\therefore \frac{1}{s^2} \propto \frac{1}{r}$, and $s^2 \propto r$.

Again, since $v \propto \frac{r}{t}$, $v^3 \propto \frac{r^3}{t^3}$. But, $r^3 \propto t^2$; $\therefore v^3 \propto \frac{t^2}{t^3}$, or $v^3 \propto \frac{1}{t}$; $\therefore \frac{1}{s^3} \propto \frac{1}{t}$, and $s^3 \propto t$.

By the law of gravity, $g \propto \frac{1}{r^2}$; $\therefore \frac{1}{l} \propto \frac{1}{r^2}$; and $l \propto r^2$. But $s^2 \propto r$; $\therefore s^4 \propto r^2$, and $s^4 \propto l$.

Bringing together these results, we find four variations, $s \propto s^1$; $r \propto s^2$; $t \propto s^3$; $l \propto s^4$.

Hence, we have the *reciprocal of velocity*, s; the *distance*, r; the *periodic time*, t; and the *reciprocal of gravity*, l; respectively denoted in their ratios by the geometrical series, s^1, s^2, s^3, s^4, in which the first term and the ratio are equal.

348. *Mode of using these variations for calculation.*—If the *velocities* of two planets are given, we first take their reciprocals, and thus have the ratio of s for the two. The terms of this ratio are then raised to the second, third, or fourth power, according as we wish to compare r, or t, or l.

But if the ratio of *distances*, or *times*, or *gravities* is given, the corresponding root is first extracted, in order to find the ratio in respect to s, and then we proceed as before.

349. *Examples.*—

1. The planetoid Pallas has a period of $4\frac{2}{3}$ years; how much further is it from the sun than the earth is? How much less is it attracted? How much slower does it move?

Let t, s, r, l be used for the earth, and T, S, R, L for Pallas

Then, $t : T :: 1 : 4.667$;

$\therefore 1^{\frac{1}{3}} : (4.667)^{\frac{1}{3}} :: s : S$;

$\therefore s : S :: 1 : 1.67$; that is, the earth's velocity is 1.67 times as great as that of Pallas.

Again, $r : R :: 1^2 : (1.67)^2 :: 1 : 2.7926$; or, Pallas is 2.7926 times as far from the sun as the earth is.

Again, $l : L :: 1^4 : (1.67)^4 :: 1 : 7.7985$; therefore, the earth is attracted by the sun about 7.8 times as much as Pallas is.

2. What would be the period of a satellite revolving about the earth close to its surface?

The distance of this satellite to that of the moon is as $1 : 60$;

$\therefore s : S :: 1 : (60)^{\frac{1}{2}}$; $\therefore t : T :: 1 : (60)^{\frac{3}{2}} :: 1 : 464.66$.

But the moon's period is 27.32 days, or 655.68 hours. Hence, the period of the satellite is 1.411 h., or 1h. 24m. 39s. nearly.

3. How much faster must the earth rotate on its axis, in order that bodies on the equator may lose all their weight?

This is just the condition of the body in example 2d, whose period is 1.411h. But the earth's time of rotation is 24h., which is 17 times 1.411h. Therefore, if the earth were to rotate 17 times more rapidly than at present, all bodies on the equator would just lose their weight, and revolve independently.

4. What would be the periodic time of a body revolving about the earth, at the distance of 5,000 miles from the center? *Ans.* 1h. 59m. $23\frac{1}{4}$s.

5. What must be the moon's distance from the earth, in order to revolve about it once in a year?

Ans. 1,344,000 miles.

6. Suppose a planet to be discovered, whose daily velocity is 5 times as great as that of Mercury, what is its distance from the sun's center? *Ans.* 1,430,000.

CHAPTER XVIII.

COMETS.—SHOOTING STARS.

350. *A comet defined.*—A comet is a body which consists of nebulous matter, and revolves about the sun in a very eccentric orbit. Most comets present a roundish ill-defined appearance, often having a bright central part called the *nucleus*. The fainter part, surrounding the nucleus, is called the *coma* (*hair*); and the *tail*, which distinguishes many comets, is merely the extension of the coma. It is the streaming appearance of the tail, resembling hair, which gave the name "comet" to this class of bodies. The nucleus has been sometimes supposed to be solid; but it probably consists always of nebulous matter in a more condensed state than the other parts. The nucleus and coma are called the *head* of the comet.

351. *Number of comets.*—Many hundreds of comets have been recorded, most of them, of course, visible to the naked eye. But lately it is observed that most comets are telescopic objects. And many which would otherwise be seen, escape observation by being above the horizon only in the daytime. The whole number, therefore, belonging to the solar system is undoubtedly to be reckoned by thousands, or tens of thousands.

352. *Eccentricity of orbit.*—All known cometary orbits are more eccentric than any planetary orbit; and most of them are

exceedingly so, their perihelion being as near the sun as Mercury and Venus, or nearer, and their aphelion as far off as the most distant planets, or even beyond. And some appear to be ellipses of infinite length—that is, parabolas; while others exhibit the form of hyperbolas. In orbits of these last forms, comets can, of course, pass the perihelion but once.

353. *Consequences of great eccentricity.*—

1. One effect of this great eccentricity is, that a comet is too far from the earth to be seen, except during a small part of its revolution, while it is near the center of the system.

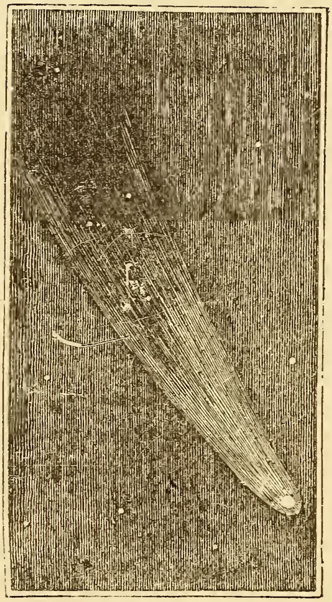
2. Another effect is, that great changes take place in the condition of the nebulous matter of which the comet is composed. As a comet approaches the sun, both the nucleus and coma grow less in diameter, and enlarge again as it departs But the tail, if there is one, is rapidly lengthened as the comet approaches, and is diminished in length when it withdraws. Sometimes, a comet, whose appearance is spherical when first seen, begins suddenly to exhibit the formation of a tail as it comes nearer, which at length stretches over a large arc of the sky; and after the perihelion passage, as it departs from the sun, the tail wholly disappears before the comet becomes invisible.

It might be supposed that this diminution of the coma results from the loss of material which is taken away to form the tail, while the comet is approaching the sun; and that the subsequent enlargement is due to the return of the same material, as the tail is contracted. But this will not fully explain the observed changes; for the contraction and subsequent expansion occur when no tail is formed. Hence, it is supposed that the heat of the sun reduces the dimensions of the nucleus by expanding a portion of it into the coma, and also changes the nebulous matter of the coma into a pure, transparent gas, which is afterward condensed into a visible form again, as the comet withdraws from the sun.

354. *Form and direction of tails of comets.*—The forms of tails belonging to different comets are exceedingly varied. In general, however, the sides diverge from the head, so that the

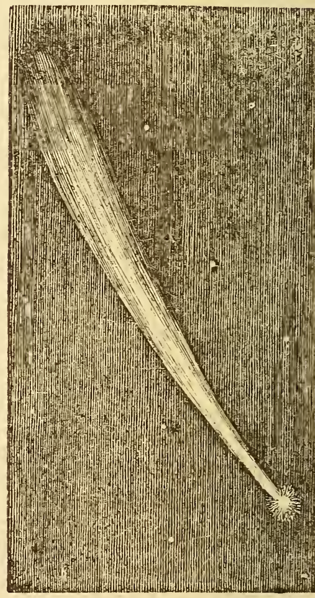
most distant and faintest part is broadest, as in the comets of 1680 and 1811 (Figs. 82, 83). In some cases, the divergency is very slight, as in the comet of 1843 (Plate I., at the end).

Fig. 82.



COMET OF 1811.

Fig. 83



COMET OF 1680.

Not unfrequently, the principal light of the tail appears to proceed from its edges, presenting somewhat the aspect of two tails diverging from the sides of the coma. In such cases, the coma and tail seem to have the form of a hollow paraboloid, so that we look through a much greater extent of illuminated matter on the sides than in the central parts. In the comet of 1858, the nucleus was at one time surrounded by a series of parabolical envelopes, which increased in number as the comet approached the sun. (Pl. II., Fig. 1).

In a few instances, the tail has been known to consist of several luminous rays, diverging from each other, as the comet of 1744, in which there were six, the extreme ones making with each other an angle of about 45°.

The general direction of the tail is *from the sun;* so that, as a comet approaches the sun, the tail follows it; but as it recedes, the tail is directed forward. The axis of the tail is not, however, a straight line, but more or less curved backward, so that the convex side of the curve is foremost in the motion.

355. *Cause of the direction and curvature of the tail.*—Modern telescopic observations on some of the most conspicuous comets, show that the material of which the tail is formed is first projected *toward* the sun, rather than from it; and that some force emanating from the sun then drives it, with great velocity, in the opposite direction, causing it to sweep past the nucleus on both sides, and stretch millions of miles into space. The rate at which it is thus driven from the sun is sometimes enormous. In the case of Halley's comet, in 1835, the nebulous matter had a velocity of 2,000,000 miles per day. In Donati's (1858), it reached the rate of 8,000,000 miles per day. What force it is which the sun thus exerts in a direction opposite to its gravity, it is vain to conjecture. It must be supposed that at least a part of the material driven so violently from the comets is dissipated and lost; and there is indication of this in the diminished size and brilliancy of those whose returns have been noticed. Perhaps the numerous comets which have no tails have been divested of them by this process.

The bending of the tail backward is a necessary consequence of the longer arc which the extreme part of the tail must describe. The material of the tail has the same velocity in the orbit as the head, when it is driven from it. This velocity it retains; but, having to describe a curve about the sun several millions of miles outside of the other, it must, of course, fall behind it.

356. *Dimensions of comets.*—The dimensions of comets are various, and, on account of their nebulous character, they never admit of accurate measurement. The nucleus of a large comet is sometimes 5,000 miles, and the coma 200,000 miles in diameter, while the tail has, in one case, attained the extraordinary length of 200,000,000 miles.

The apparent length of a comet's tail is often sufficient to

span an arc of 20° or 30° on the sky, and sometimes much more than this. The comet of 1680 extended 97°, and that of 1861, 106°. The fainter part, in all cases, is seen only by indirect vision.

It is obvious that the real length can not be inferred from the apparent, until the distance from us, and the obliquity to our line of vision, are obtained.

357. *Light of the comets.*—These bodies, like the planets and satellites, shine by solar light which they reflect to us. But, unlike all planetary bodies, they are in a condition so attenuated, that the sun's rays penetrate every part of them without obstruction. The brightness of a star is not diminished in the least when seen through the tail or coma of a comet. In a few instances, a star has been seen through the nucleus, and even then was not essentially dimmed.

A satisfactory proof that the comets are seen by the sun's light which they reflect, is, that their brightness diminishes as they recede from the sun; so that they are at length lost to view, not by being too *small* to fill an appreciable angle, but too *faint* to be visible. This would not be true of a self-luminous body: its brightness would remain the same at all distances from us; that is, its light would diminish no faster than its apparent area. Appendix G.

358. *Quantity of matter in comets.*—Though some of the largest comets surpass all other bodies in the solar system in *magnitude*, yet in respect to their *mass* they are too small to have produced as yet the slightest perceptible effect. They sometimes come very near planets and their satellites, but are never known to exert the least influence on them. They do, of course, attract the planets, because they are attracted by them, and suffer great disturbances from them. But until they themselves produce some effect which is appreciable, their mass must be regarded as infinitely small.

359. *Directions of cometary motions.*—The cometary orbits are unlike the planetary, not only in the degree of their eccentricity, but in the varied positions of their planes. Instead of

being limited to a narrow zone like the zodiac, they make every variety of angle with the ecliptic, so that a comet is as likely to pass round the sun from north to south as from west to east. And whether the orbit is much or little inclined, the comet's motion in it is as often retrograde as direct.

360. *Means of determining a comet's orbit.*—Since a comet can be seen only during the time of its describing a short arc near the perihelion, the astronomer has not the same opportunity for fixing its orbit as he has in the case of a planet, which can be observed in all parts of its course.

It is true in theory, that by any three observations on the position of a body revolving about the sun, its whole orbit can be determined. But if it is very eccentric, and the observations are confined to a small portion at one extremity, the slightest error may greatly change the distance of the aphelion, and consequently the length of the axis and the periodic time.

It is usual, therefore, to assume the path to be a parabola—that is, an ellipse of infinite length, whose eccentricity is 1. There are then but *four* of the seven elements (Art. 325) to be determined—namely, the 3d, 4th, 6th, and 7th. But instead of the 2d element in Art. 325, there may be substituted the perihelion distance, making in all *five* elements, as follows:

2. The perihelion distance.
3. The longitude of the ascending node.
4. The inclination of its orbit to the ecliptic.
6. The longitude of the perihelion.
7. The place of the body at a given time.

These five elements may usually be determined without much difficulty.

361. *Process of finding the five elements.*—The right ascension and declination of the comet are observed on every favorable night with all possible care, and the exact time of every observation is recorded. Three of these dates are selected, several days apart; and from the right ascension and declination at each date are deduced the geocentric longitude and latitude of the comet. The heliocentric places of the earth at the same times are known, since each is 180° from the

sun's apparent place at the same time. If we imagine three straight lines to be drawn from the known places of the earth through the corresponding positions of the comet, its distance from us in each line must be determined by the following conditions, in accordance with Kepler's laws:

1. A plane passing through the three positions must also pass through the sun.
2. The three places must be in a parabola, whose focus is at the sun.
3. The areas included between the radii vectores drawn to the sun must be proportional to the times.

Points are successively assumed in the given lines, until at length those are found which will fulfill the above conditions. By this tentative process the orbit is approximately determined.

Three other dates may then be tried in the same manner; and if the results nearly agree, the mean may be considered more accurate than either.

This method is just as applicable for determining, approximately, the orbit of a newly discovered planet or planetoid. But in these cases, the observations can usually be followed up in various parts of the orbit, and thus previous errors corrected.

362. *Determination of the remaining elements.*—If a comet remains in sight for several months, it is quite probable that long-continued and careful observations will show that the orbit is not truly a parabola, but an ellipse. In such a case, the 1st and 2d elements (Art. 325) may be computed, and the 5th corrected. Though there may be good evidence that the orbit is an ellipse, yet there must be great uncertainty in any determination of periodic time, mean distance, and eccentricity, until they are settled conclusively by a return of the comet to its perihelion.

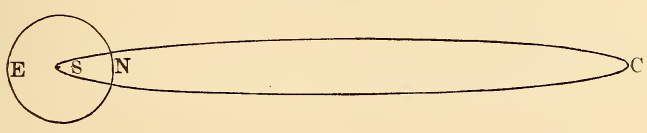
A comet, on its return to its perihelion, is not to be identified so much by its physical aspect, as by the agreement of its elements. If the place of the node, the inclination to the ecliptic, the place of the perihelion, and the perihelion distance agree very nearly with those of some comet, which has been previ

ously seen, it is fairly presumed to be the same, even though its appearance may essentially differ.

When a comet is thus identified, its periodic time is, of course, known; and from this, its mean distance, and the eccentricity of its orbit are readily obtained.

363. *Comets whose elements have been computed, but not verified.*—The orbits of more than 300 comets have been computed. But a great majority of these appeared to be parabolas, and no prediction of their return could be made. In about 60 cases, the movement of the comet seemed to afford evidence that the orbit was an ellipse, and in 7 others, a hyperbola. For the elliptic orbits, the returns were, of course, predicted. But most of the computed periods are long, generally hundreds, and, in several instances, thousands of years; so that as yet very few of them have been verified. The period of a comet seen in 1849 was calculated to be 2,115 years. If the computation be supposed correct, the distance of this comet from the sun at aphelion is about 11 times the distance of Neptune, and its next return will occur in the year 3964. But the results of calculation, in such cases, are exceedingly uncertain. The circle NE (Fig. 84) represents the orbit of Neptune; S, the sun; and SC, the orbit of the comet of 1849. The focus is very close to the vertex of the ellipse, as represented by the dot

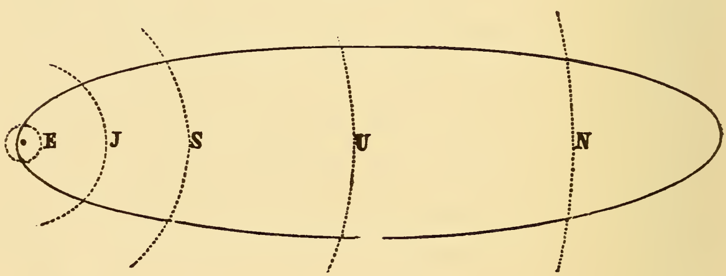
Fig. 84.



364. *Halley's comet.*—This is the only comet of long period, the elements of whose orbit are all known. It is a comet of considerable splendor, and describes its orbit in 75 or 76 years. Halley observed it in 1682; and finding, by computation, that its path was nearly identical with those of the comets of 1607 and 1531, he conjectured that the three were one and the same comet, and predicted that it would return early in 1759. It did return March 12th of that year, and again on the 16th of

November, 1835. On its last return, it reached the perihelion within two days of the calculated time. What renders such agreement remarkable is not merely the great length of the periodic time, but the great allowance to be made for the disturbing influence of the principal planets. On its last return but one, the period of this comet had been increased nearly two years by the attractions of Jupiter and Saturn. The aphelion is nearly 600,000,000 miles beyond the orbit of Neptune. Figure 85 presents the form of the orbit of Halley's comet, and its magnitude in comparison with the larger planetary orbits. The eccentricity is nearly 0.97 ; hence the distance of the focus from the vertex is only .03 of the semi-major axis. The dot at the left hand correctly represents the place of the focus occupied by the sun.

Fig. 85.



365. *Comets of short period, whose orbits are known.*— The eight comets in the following table are known by the names of the persons who either discovered them, or first predicted their returns.

COMET.	Period in years.	Perihelion Distance.	Aphelion Distance.
Encke's.............	3½	32,000,000	387,000,000
De Vico's...........	5½	110,000,000	475,000,000
Winnecke's.........	5½	70,000,000	510,000,000
Brorsen's...........	5½	64,000,000	537,000,000
Biela's..............	6½	82,000,000	585,000,000
D'Arrest's..........	6½	108,000,000	530,000,000
Faye's...............	7½	192,000,000	603,000,000
Mechain's..........	13½		

It will be perceived by the table, that these comets considerably resemble each other in period and distance. They are also alike in being telescopic, and nearly or entirely destitute of tails, and in moving from west to east, excepting the last, at inclinations to the ecliptic not larger than 15° or 20°.

366. *A resisting medium.*—Two of the above comets, Encke's and Faye's, have given decided indications of acceleration in their orbits. This shows that they meet with some obstruction, which diminishes their projectile force, in consequence of which the centripetal force draws them into a smaller orbit, which is, of course, described in less time.

Some suggest that as the received theory of light requires the existence of a medium throughout space, a substance of so little density as a comet may possibly be obstructed by it sufficiently to render the diminution of period perceptible.

According to others, the obstruction may arise from collision with innumerable small bodies which revolve about the sun. The earth is meeting with such bodies incessantly, as is proved by the numerous shooting stars which are continually striking into the atmosphere. It is reasonable to suppose that other bodies, as well as the earth, also meet them; and that the thinnest and lightest bodies, such as the comets, should show the effect of such collisions.

If Encke's and Faye's comets, in either of these ways, are gradually diminishing their periodic times, then every other comet must by and by exhibit the same change; and the time will come eventually when all this class of bodies will, at their respective perihelia, approach so near as to fall upon the sun, and be combined with its substance.

367. *Division of Biela's comet.*—One of the most remarkable facts which has occurred in the history of comets was the division of Biela's comet into two distinct comets. This appearance was first noticed at its return, in 1846. The two comets were unequal in size, and the larger had a short tail; the other was only a little elongated; whereas, before the division, the comet was spherical, without any appearance of a tail. Since the division took place, the two bodies have moved

in separate and independent orbits. Their distance apart in 1852 was about $1\frac{1}{2}$ millions of miles. Appendix H.

368. *Other remarkable comets.*—A few other comets are here mentioned, which, on account of their splendor, or for some other reason, are regarded as objects of special interest.

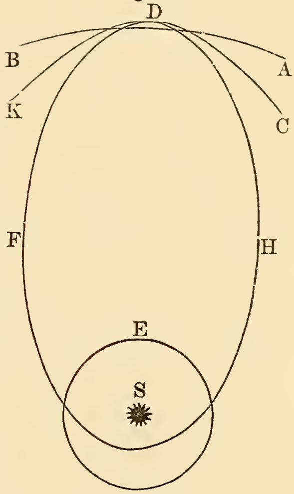
369. *The comet of* 1680.—(Fig. 83). This was a comet of unusual brilliancy, and appears to have been the first whose elements were calculated by Newton. At perihelion, its center was only 130,000 miles from the surface of the sun; so that, if its diameter was as large as that of many comets, it must have come in contact with it. Its velocity at perihelion was sufficient to have carried it round the sun at that distance in less than three hours.

370. *The comet of* 1744.—This was the most splendid comet of the 18th century. The remarkable features of it were, its great brightness, and the number of its tails. Its light was nearly equal to that of Venus, and it was distinctly seen in the daytime, even by the naked eye. After passing the perihelion, its tail was spread into six distinct branches, near 40° in length, and the extreme ones diverging about as many degrees from each other.

371. *The comet of* 1770.—The great interest which attaches to this comet arises from the fact that it has twice suffered a great change of orbit, in consequence of the disturbing action of Jupiter. It first appeared in 1770, shining with considerable splendor. In 1776, it again passed the perihelion, and has never been seen since. Computations made by La Place and others showed that, before its first appearance, it had revolved in a large orbit, beyond our vision, and had a period of 48 years. In 1767, it came near Jupiter, and lost so much of its velocity, that it was drawn into a small orbit, whose perihelion was far within the orbit of the earth. As two of its revolutions were about equal to one of Jupiter, it was predicted that it would be again subjected to a great disturbance at its aphelion. This actually took place in 1779, so that it has

never returned to our view. Its present period is calculated to be about 20 years. AB (Fig. 86) is a part of Jupiter's orbit; E is the earth's orbit; CD is the path of the comet before 1767. Near D, it was so retarded by Jupiter, that the sun drew it into the small orbit DFH, which it described twice, when, near D, it was again powerfully affected by Jupiter, and received a great acceleration, which caused it to pass out once more into a large orbit, DK.

Fig. 86.



372. *The comet of* 1843.—The brightness of this comet was so great, that it was seen during the day. Its perihelion distance was less than 550,000 miles, and its exterior parts were probably in actual contact with the sun. No other comet has been known to approach so near. The tail spanned 70° of the sky, and was unusually straight and slender, as exhibited in Pl. I.

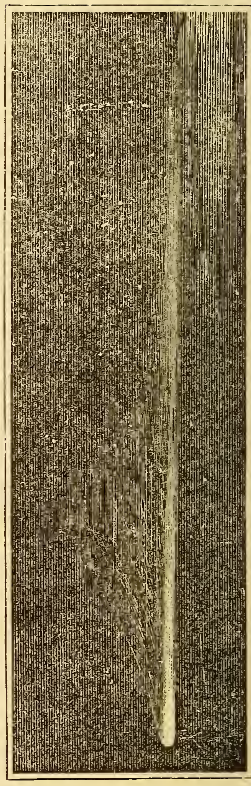
373. *The comet of* 1858.—This is also called Donati's comet, having been discovered by Donati, of Florence. It was remarkable for the series of envelopes formed successively about the nucleus, as it approached the perihelion. The appearance of the head is shown in Pl. II., Fig. 1, and the entire comet in Fig. 2. Its period is computed to be about two thousand years.

374. *The comet of* 1861.—This comet came so near the earth, that it is believed a part of the tail swept across it. But it is not certain that any visible effect was produced. The apparent length of its tail, at one time, was 106°. Fig. 87 shows its form at that time.

375. *Effects of collision between a planet and a comet.*—Whether a direct collision between the earth and the nucleus

of a comet would produce serious effects, it is impossible to know, because so little is understood respecting the density of the nucleus. But the coma and tail consist of matter thousands of times more rarefied than the earth's atmosphere, and would probably fail to penetrate it at all. The earth is thought to have passed through a comet's tail, at least in one instance, but without producing any perceptible effect.

Fig. 87.



376. *Shooting stars.*—This is the popular name given to those bodies which appear like stars or planets moving across some part of the sky, and then vanishing. They are equally well known by the name of meteors. They may be seen in any clear night, by watching an hour or two, especially if the moon is not shining.

377. *Height and velocity.*—By means of concerted observations, made at stations quite distant from each other, the angle can be measured, which is included by lines drawn from a meteor to the stations, both at the beginning and end of its motion, and thus its distance and velocity can be measured. The heights of meteors are thus found to be generally about 50 miles, and their velocities 20 or 30 miles per second. Coming into the air with such great velocity, they are almost instantly set on fire, and their substance becomes incorporated with the atmosphere.

378. *Gaseous meteors.*—If the ordinary meteors were more dense than a gas, they would hardly lose all their motion, as they do, before reaching the earth. The most interesting facts relating to this class of bodies are the following:

1. They often occur in showers—that is, thousands and hundreds of thousands of them are seen in a single night.

2. These showers have periodical returns.

3. The meteors of a shower come into the atmosphere in a given direction, or, in other words, in parallel lines. The optical effect is, that they appear to describe arcs of great circles, having a common place of intersection.

379. *Dates of meteoric showers.*—The most remarkable meteoric shower of the present century was November 12–13, 1833. Not less than 200,000 meteors were seen during the night at any one station. Like showers occurred at the same time in 1799 and 1866. And generally, there are more meteors about the 12th of November than at any other time of the year.

Other dates at which meteors are unusually abundant are April 21st, August 10th, and December 7th.

380. *Origin of the gaseous meteors.*—The known motion of the earth, and the observed velocity and direction of this class of bodies, lead to a knowledge of their heliocentric motions. It is found in this way that they describe ellipses about the sun, and are therefore to be regarded as minute cometary bodies. Those which come in showers seem to belong to extensive groups, which revolve about the sun in zones or rings. There appear to be three or four of these zones, whose planes are situated at different obliquities to the ecliptic, and across which the earth passes once a year. When the earth traverses a more crowded portion of such a ring of meteors, the phenomenon of a meteoric shower occurs. Appendix I.

381. *Solid meteors.*—There is another class of meteoric bodies, which afford indubitable evidence of being solid. Like the gaseous meteors, they plunge into the atmosphere with great velocity, and are inflamed by the violent attrition. Before reaching the earth they usually explode, and scatter their fragments. Some of them, however, appear to lose only small portions of their mass by explosion, and pass on in their orbits around the sun—greatly disturbed, of course, by the earth's attraction.

13

382. *Aerolites.*—This is the name usually given to the fragments thrown down by solid meteors; though in rare instances, an aerolite obviously constitutes the entire meteor itself. Aerolites consist of iron, silex, and a few other materials, which are all known among terrestrial substances. But they are always distinguishable from terrestrial bodies by their peculiar structure. Since the great velocities of meteors, solid as well as gaseous, have become known, the former theories as to the origin of meteoric stones, or aerolites, have been abandoned. Such velocities, if they could be generated at all on the earth, could never exist in horizontal or downward directions. Both solid and gaseous meteors are therefore considered as describing orbits about the sun. The interplanetary spaces, which have been generally reckoned as vacant, may perhaps be to a great extent occupied by innumerable bodies, of a grade far below that of comets and planetoids.

CHAPTER XIX.

THE FIXED STARS.—THEIR CLASSIFICATIONS.—THEIR DISTANCES AND MOTIONS.—DOUBLE STARS, CLUSTERS, AND NEBULÆ.—THE NEBULAR HYPOTHESIS.

383. *The stellar universe.*—The bodies described in the foregoing chapters all belong to the solar system. If our investigations are extended outside of this system, we find that there are other systems, greater or less than this, unlimited in number, and separated from the solar system and from each other by solitudes so vast, that each system is only a point in comparison with the distances between them. The central sun in each of these countless systems is a fixed star.

The word "universe" is employed to express the sum total of all these systems, the number of which, and the extent of space occupied by them, are utterly beyond the reach of human comprehension.

384. *The fixed stars, and their magnitudes.*—The fixed

stars are so called, because, to common observation, they always maintain the same situations with respect to each other. All the thousands of bright points ordinarily seen in the sky by night are fixed stars, with the exception of two or three, possibly four, which are planets.

The fixed stars are classified according to *magnitudes*, though the word, when thus used, signifies only *degrees of brightness.* The stars which can be seen by the naked eye, in the most favorable circumstances, are divided into six magnitudes. Those which can be seen only by the aid of the telescope, called *telescopic* stars, are arranged into several more; so that all the magnitudes are 16 or 18.

Stars of the same magnitude are not equally bright; for there is a continual gradation in respect to brightness; so that, if the intensity were accurately measured, probably the light of but very few would be found exactly equal.

Stars of the first magnitude are fewest in number, and, generally, the smaller the magnitude, the larger the number of stars included under it. The limits of the successive magnitudes differ somewhat, according to different astronomers; but the following round numbers do not vary widely from any of them.

1st magnitude . .	20	4th magnitude . .	300
2d " . .	40	5th " . .	950
3d " . .	140	6th " . .	4450

In all, near 6,000, visible to the naked eye. The numbers of the telescopic stars increase at so rapid a rate, that they have to be reckoned by millions.

385. *Cause of unequal brightness.*—We might suppose either that the stars are themselves unequal in respect to the quantity of light which they emit, or that they appear unequally bright on account of their different distances. It is undoubtedly true that there is some diversity in the bodies themselves; and yet, the rapid increase of numbers as the magnitudes are less, indicates that difference of distance is the chief cause of inequality in brightness. If there is any approach to

a uniform distribution of the stars in space, those which are nearest should be fewest in number, and should, in general appear brightest.

386. *Constellations.*—The fixed stars are also classed topographically in constellations. This division is very ancient; and some of the constellations are mentioned by the earliest writers. The names given to them are those of the animals, heroes, and other objects of pagan mythology.

Constellations of the zodiac.

Aries.	Libra.
Taurus.	Scorpio.
Gemini.	Sagittarius.
Cancer.	Capricornus.
Leo.	Aquarius.
Virgo.	Pisces.

Constellations north of the zodiac.

Ursa Major.	Auriga.	Cygnus.
Ursa Minor.	Leo Minor.	Vulpecula.
Draco.	Canes Venatici.	Aquila.
Cepheus.	Coma Berenices.	Antinous.
Cassiopeia.	Bootes.	Delphinus.
Camelopardalus.	Corona Borealis.	Pegasus.
Andromeda.	Hercules.	Ophiuchus.
Perseus.	Lyra.	

Constellations south of the zodiac.

Cetus.	Monoceros.	Hydra.
Orion.	Canis Major.	Crater.
Lepus.	Canis Minor.	Corvus.
Centaurus.	Crux.	Eridanus.
Lupus.	Argo Navis.	

The foregoing are the principal constellations; but several more, mostly small ones, may be found on globes and charts.

Within each constellation, the brightest stars are designated by the letters of the Greek alphabet in the order of brightness. Thus, α Lyræ, is the brightest star in Lyra; β Scorpionis, the brightest but one in Scorpio, etc. After the Greek letters are all used, Roman letters, and then numerals, are employed. In some cases, the order of brightness does not accord with the order of the alphabet. This may result from a change of brightness, which has taken place since the stars were first named. When a capital letter follows a number, there is reference to the catalogue of some astronomer. Thus, 84H is the star 84 of a certain constellation in Herschel's catalogue.

A few conspicuous stars are still known by individual names given to them in ancient times; as Arcturus, Antares, Sirius, Vega, etc.

The first catalogue of stars was made by Hipparchus, before the time of Christ, and contained 1,022 of the most conspicuous stars. Catalogues of the present day contain hundreds of thousands of stars, whose right ascensions and declinations are given for a certain date.

387. *Effect of telescopic power on fixed stars.*—One indication of the vast distance of the fixed stars is, that no power of a telescope sensibly magnifies them. Even under a power which increases the diameter of a body 5,000 times, they appear no larger than to the naked eye. It is inferred that they fill an angle so small, that 5,000 times that angle is still too minute to be perceived. Any appearance of *disk* which a star presents, either with a telescope or without, is the effect of the light upon the retina of the eye. It is called a *spurious* disk, since an increase of magnifying power causes no increase of its diameter.

388. *Annual parallax.*—Another proof that the fixed stars are at an immense distance from us, is the fact that while we shift our position every six months from one side of the earth's orbit to the opposite, a distance of 185,000,000 miles, there is no perceptible change in the relation of the stars to each other. It is only after long-continued and most accurate observation,

that a few stars have been discovered to suffer an annual change of position, which is clearly of the nature of parallax.

The annual parallax of a star is the angle, at the star, subtended by the radius of the earth's orbit. As this angle is in almost all cases too small to be detected, it shows that the earth's orbit, seen from the distance of the stars, appears as a mere point.

389. *The parallactic path of a star.*—If the annual parallax of a star is in any case perceptible, its apparent movement during the year depends entirely on its situation in relation to the ecliptic.

A star *in* the plane of the ecliptic will appear to oscillate back and forth in a straight line once in a year. It will appear stationary at the two opposite seasons, when the earth is going toward it, and from it; and if we imagine a diameter of the earth's orbit joining these two positions, the star will seem to describe a straight line parallel to that diameter, its motion during each half-year being opposite to the general direction of the earth's motion.

But if a star at the pole of the ecliptic should exhibit any parallax, its apparent motion would be in an orbit parallel to the earth's orbit, and similar to it: it may be regarded, therefore, as a circle described about the point in which the star would be seen from the sun. Moreover, the star's apparent place, and the earth's real place in their respective orbits would be diametrically opposite.

At a point between the plane of the ecliptic and its pole, the parallactic orbit would be an ellipse, the ratio of whose axes would depend on the latitude of the star.

390. *Discovery of annual parallax.*—It is justly reckoned among the greatest achievements in practical astronomy, that the annual parallax has, in a few cases, not only been clearly detected as existing, but has been satisfactorily measured, though it is never so great as 1″.

The parallax of α Centauri is 0″.91; that of 61 Cygni, 0″.35; of α Lyræ, 0″.26; of Sirius, 0″.23. A few others have been

obtained, which are still smaller, and therefore less reliable.

The parallax of a star is most satisfactorily determined, when it is in the same telescopic field with other stars. For then the distances between the stars may be measured with great precision by a micrometer, and all errors arising from aberration, refraction, and instrumental disturbance are wholly avoided, because all the stars in the same field are affected alike by these causes of displacement. Parallax is the only circumstance which can produce an annual change in their relative positions. The star 61 Cygni is, in this respect, very favorably situated, and its parallax is thought to be quite accurately determined.

391. *Distances of those stars whose parallax is known.*— If a triangle is formed by the lines joining the sun, earth, and star, and the angle at the sun be a right angle, we have the proportion

Sin an. par. : rad : : 92,381,000 miles : dist. of the star.

This gives the distance of α Centauri, the nearest star, 21,000,000,000,000 miles, nearly. Light, moving at the rate of 185,000 miles per second, would require about 3.6 years to come from that star to us; 9.3 years from 61 Cygni; 12.6 years from α Lyræ; and 14.2 years from Sirius. And if we reckon the parallax of the pole-star at $0''.07$, as it has been computed to be, it requires 47 years for its light to reach us.

In order to compare these amazing distances with the dimensions of the solar system, we may use with advantage the diagram described in the note, Art. 263. The distance from the sun to Neptune being represented by 30 feet, the distance of the nearest star, α Centauri, must be represented by 40 miles, and that of 61 Cygni by 110 miles, etc. Thus isolated are the systems of the universe from each other.

As to all other stars besides those above named, it is only known that they are still more distant. There is no improbability that, from the remotest telescopic stars yet seen, light may occupy thousands of years in coming to us. Therefore, we see all the stars as they were years ago, perhaps not as they are now. And if at any time a change has been detected

in the aspect or place of a star, that change occurred, not when it was seen, but 10, 100, or 1,000 years before, according to its distance.

392. *Nature of the fixed stars.*—The stars are situated at such vast distances from the solar system, that if they merely reflected the light of the sun, they would be invisible. In order to exhibit such brightness as they do, they must not only shed light, but a very intense light of their own. They can not be compared with any one of the bodies in the solar system, except the sun itself. All the fixed stars, therefore, are to be considered as suns, and probably the centers of systems resembling the solar system. It is ascertained, respecting some of those stars whose distance is known, that they shed more light than the sun. For example, α Centauri has been found to shed near *four* times as much light as the sun. For the light of the sun at the earth is about 500,000 times as great as the light of the full moon. And the light of the full moon was found by Sir John Herschel's observations to equal 27,000 times that of α Centauri. Therefore, the light of the sun at the earth is (500,000 × 27,000) 13,500,000,000 times that of α Centauri at the earth. But that star is 230,000 times as far off as the sun. And since the quantity of light received from a luminous body varies inversely as the square of the distance, if α Centauri were brought as near to us as the sun, its light would be 52,900,000,000 $(= 230,000)^2$ times as great as it is at present, or nearly four times as great as the light of the sun.

In a similar manner, Sirius, the brightest, but not the nearest fixed star, is found to shed 100 times as much light as the sun.

On the other hand, if the sun were removed from us to the nearest fixed star, its apparent diameter would be only $\frac{1}{120}''$, and, therefore, would be a star having no sensible magnitude, and having only $\frac{1}{6}$ of the brightness of Sirius. Appendix J.

393. *Proper motion of the stars.*—There is increasing evidence that there is among the stars a parallactic motion of a higher order than the annual parallax already noticed. The entire solar system appears to be moving toward a certain

point in the constellation Hercules, whose right ascension is 260°, and its declination 35° north. This motion of the system is inferred from what is termed the *proper motion* of the stars. Since the time of Hipparchus (130 B. C.), Sirius, Arcturus, and Aldebaran have changed their position southward more than half a degree. The star 61 Cygni moves 5″ each year, μ Cassiopeiæ 4″, and ε Indi 8″; and a large number of other stars have a small progressive motion. The general effect of a motion of our own system would be to cause a minute apparent separation of the stars in the region *toward* which we are moving, and a crowding together of the stars in the region *from* which we move. From a comparison of the proper motions of several hundreds of stars, a motion of the solar system in the direction named above has been deduced. And the rate of that motion has been estimated to be about 154,000,000 miles per year, which is only one-fourth the earth's velocity in its orbit.

If the motion is really perceptible, it is probable that a *change* of direction will, after a few centuries, manifest itself, from which something may be inferred as to the position and magnitude of the orbit which the sun describes.

Some of the stars have a proper motion, which can not be explained by the supposed motion of the solar system. In those cases, it must be concluded that they are themselves describing vast system-orbits about some distant center. Appendix K.

394. *Double stars.*—It is discovered in a great number of instances that a fixed star, when examined by the telescope, really consists of two stars, very close to each other. If the distance between them does not exceed 32″, such stars are called *double* stars. Their distance apart is often less than 1″, and some are so close, that the highest power of the telescope and the most acute vision are requisite to separate them. Hence, certain double stars are habitually used as tests of the excellence of an instrument.

When Sir William Herschel first began his observations on this class of objects, in 1780, he knew of only *four;* but he extended the list to 500 himself, and the number now known exceeds 6,000.

395. *Relative intensity and color.*—In comparatively few instances are the two stars equally bright. They sometimes differ so little as to fall within the limits of the same magnitude; but generally they are of different magnitudes. Thus, the component stars of γ Leonis are of the 2d and 4th magnitudes; of η Lyræ, 4th and 8th; and of the pole-star, 2d and 9th. Figures 1, 2, 3, and 4, in Pl. III., present the telescopic appearance of the double stars there named. In 4, they are so close as to appear like a single star, of tapering form.

A fact of great interest, in relation to double stars is, that they often differ in *color*. Sometimes these colors are *complementary;* that is, they are such as would compose white light, if mingled together. In such cases, if the stars differ much in magnitude, the appearance of color in the fainter star may be only an illusion. But this can not be true when the colors are not complementary. The components of γ Andromedæ are orange and green; of χ Bootis, white and violet; of α Herculis, yellow and blue; and of β Scorpionis, white and blue.

Single stars are frequently of a deep red color; but a decided case of green or blue is never met with, except in a component of a double star.

396. *Two ways in which stars might appear double.*—The two stars which compose a double star may be supposed either to be *really* near each other, or only to appear near together, because they fall almost into the same line of vision, while one is actually at an immense distance beyond the other. In the latter case, the stars are said to be *optically* double. When Sir William Herschel commenced examining double stars, he very naturally supposed that, in the very few cases known, one star happened thus to be nearly in the same visual line with the other; and he began the work of observing them, with the expectation of detecting annual parallax in objects so favorably situated. For, if the nearer star is perceptibly affected by parallax, it would exhibit an annual motion relatively to the more distant star, in a manner not to be mistaken.

397. *Binary stars.*—It soon became evident, however, that double stars are too numerous to allow the supposition that

their apparent proximity is only casual. It was calculated that the chance, that of all the stars visible to the naked eye, two would accidentally appear within 4″ of each other, was only 1 in 9,000; whereas one hundred such cases were already known.

But another most interesting discovery was presently made; namely, that some of the double stars exhibit motions which indicate a revolution of one around the other—or, rather, of the two around a common center, and in periods of various lengths, having no connection whatever with the earth's annual motion. Such motion can not be parallactic; it must be real; and such stars are not optically, but *physically* double. They are called *binary stars*, and are to be regarded as the centers of double stellar systems.

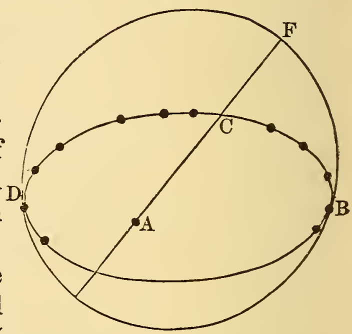
398. *Gravitation outside of the solar system.*—The binary stars afford evidence that the same law of attraction which prevails within the boundary of the solar system prevails also at immeasurable distances beyond it. In the case of every binary star which has yet completed the whole, or any considerable part of its revolution, since its discovery, it is found that the path of one component star is an ellipse, while the other occupies one of the foci within it. Hence, the law of attraction is, *gravity varies inversely as the square of the distance*, just as within the solar system. Though the relative motion may be represented by considering either star as occupying the focus, and the other star as revolving about it, yet the true focus is the center of gravity between them, while each describes its orbit about that center.

399. *The real and the apparent orbit.*—It is not to be assumed that the plane of a stellar orbit is perpendicular to our line of vision. But if it is oblique, although it is always projected on the sky as an ellipse, yet the apparent eccentricity may differ in any degree from the real eccentricity, and the central star will probably appear out of the focus of the apparent orbit. The true orbit, however, can be readily deduced from the apparent one, by means of the position of the central star. If the plane of revolution of a binary star were coinci-

dent with our line of vision, one star would appear to oscillate in a straight line across the other.

The ellipse, BCD (Fig. 88), represents the apparent orbit of ξ Ursæ Majoris, the central star being at A. The real orbit, of which A is the focus, is BDF.

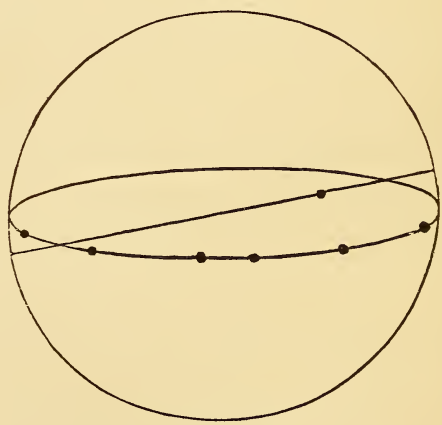
Fig. 88.



The apparent orbit of α Centauri is still more eccentric (Fig. 89), compared with the real one, because more oblique to the line of vision. It has not yet described quite half its orbit, since it began to be observed.

At the bottom of Pl. III. are shown the relative positions and distances of γ Virginis from 1837 to 1860, and the form of the apparent orbit. The real orbit is even more eccentric, the major axis being somewhat foreshortened by obliquity.

Fig. 89.



400. *Periods of binary stars.*—The shortest period known is that of ζ Herculis, about 31 years. The period of η Coronæ is 43 years; that of ξ Ursæ Majoris (Fig. 88) is 58 years. These, and a few others of short period, have completed their

revolutions once or twice since they were discovered. The orbits of such are quite accurately determined. One revolution of α Centauri (Fig. 89) has not yet been made since its discovery; its period is calculated to be 77 years. A large number of binary stars, whose periods are computed to be some hundreds or thousands of years, have been observed as yet only through a short arc; hence their periodic times, and the forms of their orbits, are quite uncertain.

401. *Dimensions of stellar orbits.*—There are two binary stars whose parallax has been so satisfactorily measured, that their distances from us may be considered as well known; these are α Centauri and 61 Cygni. Hence, by the angular length of the semi-major axes of their orbits, we may find the mean radius vector of each. The major axis of the orbit of α Centauri is about 30″, and its distance from the earth is 21,000,000,000,000 miles.

∴ rad : sin 15″ : : 21,000,000,000,000 : 1,464,000,000 miles; which is equal to about 16 times the earth's distance from the sun. The distance between the components of 61 Cygni is about 4,012,000,000 miles.

402. *Masses of the binary stars.*—For those binary stars whose periods and distances apart are known, the mass of the system can be computed. For $M \propto \frac{D^3}{P^2}$; hence, for a Centauri (the earth's distance from the sun, and its period being called 1), $M = \frac{16^3}{77^2} = 0.69$. That is, the mass of the two components of a Centauri is about 0.7 of the mass of the sun and earth. So, for 61 Cygni, whose period is computed to be 540 years, and the distance of the two components 44 times the radius of the earth's orbit, the mass of the double star is 0.3 of the mass of the sun and earth.

403. *Triple and quadruple stars.*—There are a few instances of three or four stars, which are known to be physically connected, and to constitute a system. Figs. 5, 6, Pl. III., present the appearance of 11 Monocerotis and ζ Cancri. In

the latter, the two close components revolve in 59 years, and the distant one more slowly. The faint star ε Lyræ is quadruple, consisting of two very close double stars. They give evidence of belonging to one system, but their revolutions are exceedingly slow.

404. *Periodic and temporary stars.*—There are among the fixed stars several instances in which there appear to be revolutions of another sort, the nature of which is not understood. Stars which exhibit these changes are called *periodic* stars. A remarkable example occurs in the star *o* Ceti. It passes through its changes of brightness in about 11 months. When brightest, it is of the 2d magnitude, and remains so for two weeks. It then diminishes during 3 months to the 10th magnitude, remains thus 5 months, and increases again during 3 months to its maximum of brightness.

Algol (β Persei) has a very short period, occupying only 2d. 20h. 48m. Its changes succeed each other with great regularity, thus:

During	2d.	14h.	0m.	it remains of the 2d magnitude.
"	0d.	3h.	24m.	diminishes from 2d to 4th.
"	0d.	3h.	24m.	increases from 4th to 2d.
	2d.	20h.	48m.	whole period.

Some of this class of stars have periods of only a few days, while in others the changes go on very slowly, and appear to require several years. The periods of some are quite uniform, and of others irregular. As accurate observations are multiplied, the number of known periodic stars is constantly increasing.

To this class probably belong those stars which are called *temporary* stars. That of 1572 is celebrated. It appeared so suddenly, and of such brilliancy, as to attract the attention of common people, and rapidly increased, till in a few weeks it surpassed Jupiter in brightness. It then faded slowly, and after about 1½ years entirely disappeared. Several other cases less marked than this are on record. And the earlier catalogues contain numerous stars which are not to be found at the present day. Undoubtedly some of these records are mistakes.

In two or three instances, it is known that the bodies were planets, not fixed stars. But in the course of coming centuries, some of the temporary stars may again become visible, and thenceforward be recognized as periodic stars.

405. *Cause of periodicity.*—The conclusion can not be avoided, that the variable magnitudes of stars, at least when they recur regularly, are the result of some sort of revolution More than this is mere conjecture. In some cases, the star may be partially dark on one side, and produce the changes by rotation on its axis. In others, there may be opaque bodies, either single or existing in groups or zones, revolving about the central star.

Newton suggested that the sudden appearance of a temporary star might be the result of a comet falling upon the central body, which was before invisible, and causing conflagration.

406. *Clusters of stars.*—The fixed stars are frequently grouped together in clusters, such as the Pleiades, in Taurus; Presepe, in Cancer; and Coma Berenices. If a telescope of low power is used, the number of stars appears greatly increased. Figure 1 in Pl. IV. gives a telescopic view of the Pleiades.

There are others which to the naked eye appear nebulous, but by the use of the telescope are plainly seen to be clusters; and in some of them the stars are so numerous as not to be easily counted. The clusters in Perseus and Hercules are fine examples. For the latter, see Pl. IV., Fig. 3; *a* is its appearance with a low power; *b* is the central part of it with a high power.

407. *Nebulæ.*—These are faint patches of light, having generally an ill-defined edge, and in ordinary telescopes presenting the same nebulous aspect which the closer clusters do to the naked eye. As the powers of the telescope are increased, many nebulæ are resolved into clusters of stars, while many others retain their nebulous appearance under every power yet employed. The number of nebulæ now known exceeds 5,400,

Their forms are exceedingly various; and in some cases they seem in this respect to be greatly changed as the telescope is improved in its magnifying and defining powers.

Since every advance which is made in the construction of instruments resolves some nebulæ into clusters of stars, many astronomers have been led to suppose that all nebulæ are clusters, only too remote to be resolved by means hitherto employed. Some facts, however, connected with this class of bodies seem to indicate that there are, in some regions of space, immense tracts occupied with nebulous matter not yet formed into stars.

408. *Varieties of form among nebulæ.*—

1. *Globular.* A large number, especially of the smaller nebulæ, present a circular outline, and grow brighter gradually from the circumference toward the center, thus suggesting the idea of a spherical form. The *nebulous stars*, so called, differ from them in that the nebulosity continues nearly uniform up to a central star. The *planetary nebulæ* have a well-defined edge, and no bright center, and therefore bear some resemblance to a planet.

2. *Elliptical.* Several nebulæ present the appearance of an oblate spheroid seen edgewise. The most remarkable example is the great nebula of Andromeda. Its length is $1\frac{1}{2}°$, and it is easily seen by the naked eye (Pl. IV., Fig. 2). The dumb-bell nebula, between Cygnus and Aquila, appears in the best telescopes to have an elliptical shape. The brightest part of it has a form slightly resembling a dumb-bell, or an hour-glass. (Pl. II., Fig. 4).

3. *Spiral.* This description of nebulæ is becoming rather numerous since the latest improvements in telescopes. Some nebulæ of very irregular shape, as formerly described, exhibit, in the best instruments of this day, delicate appendages having a spiral arrangement. The whirlpool nebula, near the tail of Ursa Major, is the most remarkable instance of this form (Pl. IV., Fig. 5). The crab nebula, in Taurus, may yet be found to belong to this class (Pl. IV., Fig. 4).

4. *Annular.* A few nebulæ have an outline nearly circular or elliptical; but appear more luminous on the edges than in

the central part. Such are called annular nebulæ. The appearance is that of a hollow sphere or spheroid; in which case we look through the greatest depth near the edges. An interesting example is situated in Lyra, midway between β and γ (Pl. II., Fig. 3).

5. *Irregular.* Besides the foregoing forms, which are all indicative of a central force, and of revolution, there are various shapes of great irregularity. None is so celebrated as the great nebula of Orion, which has been a subject of observation and record for more than two centuries. It becomes more extended and more complex with every new improvement in telescopes.

409. *Magnitude of clusters and nebulæ.*—Every cluster of stars, whether a complex system of suns or not, must occupy an immense space. They are at least as far distant as the nearest star, and how much further we can not know, and yet they fill a sensible angle, and some of them a large one. It is easy, therefore, to assign the lowest limit for their dimensions. The length of the nebula in Andromeda is $1\frac{1}{2}°$. Supposing it as near as α Centauri, its absolute length must be 6,000 times the distance from the earth to the sun. And if it be many times further from us than the nearest star, which is far more probable, then its dimensions must be just so many times greater.

410. *Changes in the nebulæ.*—In repeated instances it has been thought that the forms of certain nebulæ had essentially altered since their discovery. But this is not certain; for it is found that the same nebula assumes a new aspect as the telescope is improved, because some of the more delicate features, which were not before noticed, are brought to view. It may be, therefore, that all apparent changes of form hitherto noticed are to be explained in this way.

But there are a few faint nebulæ, which are known to have grown more dim within a short time; for they can not now be seen by the same instruments which only a few years ago brought them distinctly into view. In one or two instances, a nebula has entirely ceased to be visible. Such

bodies may, perhaps, have regular changes, like the periodic stars. Appendix L.

411. *The galaxy.*—This is a belt or zone, of nebulous appearance, which encircles the heavens, nearly coincident with a great circle, and cuts the plane of the equator at an angle of 63°. It is usually called the *milky-way.* Near the constellation Cygnus, it divides into two parts, which continue separate nearly a semicircle (150°), and then reunite. Its edges are generally ill-defined, and also quite crooked and irregular, having many projections and indentations.

The telescope shows that the whiteness of the galaxy is due to unnumbered stars, too faint to be seen individually. Their distribution is quite unequal; the stars, in some parts, being crowded very closely together, while here and there spaces occur which contain but few. These inequalities are most marked in the southern hemisphere. A small portion of the southern galaxy is shown in Pl. III. In the most luminous parts, Sir William Herschel estimated that, within an area less than $\frac{1}{300}$ part of the hemisphere, there passed the field of his telescope 50,000 stars, large enough to be distinctly seen. The whole number of stars in the milky-way is to be reckoned by millions.

It appears, therefore, that by far the largest part of the stars which are within the reach of our vision lie in a thin stratum or ring, in the plane of which the sun is situated. As we ourselves, being near the sun, are in this plane, we see the stars mostly crowded into the zone or belt which is called the galaxy, while over the other parts of the sky they are more sparsely distributed.

412. *The nebular hypothesis— What it proposes.*—The hypothesis which is known by the name of the nebular hypothesis proposes to explain in what manner the bodies composing the solar system may have arrived at their present state, as to motion, condition, and mutual relations, through the operation of known laws, which the Creator has employed during the almost countless ages since the material was at first formed.

413. *Argument from analogy.*—The organized bodies on

the earth, whether animal or vegetable, are not created in their mature and perfect state, performing at once all the functions for which they were designed; but they *grow* to this condition by a series of changes, which extend generally through a number of years.

So the soils of the earth were not first formed in their present condition, fitted to sustain the vegetation which clothes them; but are the result of slow disintegration of the rocky mountain tops, through the action of water and changes of temperature.

It is more in accordance with the Creator's plan of operation, so far as we can discover it, that the sun, planets, and satellites should have been brought into their present condition through a long-continued course of change, than that they should have been created and set in motion as we now see them.

414. *Facts in the solar system which form the basis of the hypothesis.*—

1. The sun, the planets, and the satellites, so far as they are known to rotate at all on their axes, rotate nearly in the same direction, from west to east. And the revolutions of all planets about the sun, and of all satellites about their primaries, with but few and trifling exceptions, are in the same general direction, from west to east.

2. The sun, which contains nearly the whole material of the system, is a sphere in a condition of intense heat. The interior of the earth is in a red-hot melted state, as is proved by the volcanoes on its surface. The moon is covered with volcanic craters, which show that it is, or has been, in the same condition, internally, as the earth now is.

415. *The nebular hypothesis stated.*—It assumes that the whole space occupied by the solar system, and extending far beyond its present limits, was filled with nebulous matter, in an exceedingly rare and intensely heated condition; and that this entire mass was put into a state of rotation in the direction which we now call from west to east.

This assumption being made, the following consequences would ensue, during the lapse of immense periods of time, in accordance with the we l-known laws of the material creation.

By gravity and the centrifugal force, the vast nebula takes a spheroidal shape.

Heat is radiated from its exterior into the boundless space around it; and by this loss, the nebula contracts in diameter. But as it contracts, the given velocity of rotation at the surface causes a quicker rate of revolution, until, at length, the centrifugal force of the equatorial part equals the attraction toward the center of the entire mass. As soon as these two forces are equal, the equatorial part rotates independently of the interior, while the latter contracts still further, and leaves the superficial part revolving as a nebulous ring.

After the central portion has left the ring, it goes on contracting as before, till it leaves a second ring. Thus, an indefinite number of concentric nebulous rings may be left, each revolving from west to east, and at a swifter rate according as it is nearer the center. The central mass, which thus successively deposits its rings, is the *sun* of the system.

416. While the material composing each ring goes on cooling and contracting, unless the quantity is exactly equal on every side, which is improbable, the whole of it, at length, is drawn toward the heaviest side, until it is gathered into a spheroid, revolving once on its own axis, while it revolves once around the central mass. These spheroids are the *planets*, revolving around the sun.

But as the planetary spheroid continues to contract by cooling, its rate of rotation is quickened, until it leaves its equatorial part revolving in a ring about it, in the same manner as the central nebula has done; and this it may do in repeated instances.

These subordinate rings are likely also to collect into so many spheroids, revolving about the larger ones, and on their own axes. These are *satellites*. In case the parts of a ring are very exactly balanced, they may preserve their condition of a ring, instead of gathering into a satellite. An example is seen in the ring of Saturn.

It is conceivable that a multitude of small rings, instead of one large one, may be detached from the central mass when the separation occurs. This seems to have been the case in

the formation of those rings from which the planetoids were formed.

417. After the planets and satellites have cooled sufficiently, they become non-luminous bodies, and are gradually changed from nebulous into a liquid or solid condition. And, in a given case, the exterior may be solid, while the interior remains in a liquid and highly heated condition. This is the present state of the earth, and the present or recent condition of the moon.

That the planes of motion throughout the system are not coincident, is to be ascribed to disturbing influences which the several bodies have been exerting on each other during the vast periods of time that have elapsed since they were detached from the solar mass.

418. *Application to other systems.*—Every fixed star which is single may be the condensed nucleus resulting from an operation similar to that which has been described; and the double and triple stars may be considered as cases in which either the nebula became divided into two or three parts, before the contraction had proceeded far, or else the nebulous mass, being very oblate, a large part of it was detached at once, and collected into a body, nearly equal to the central part.

The nebulæ of regular form, not capable of being resolved into separate stars, may still be in the condition of the solar system before its rings began to be separated from the original body.

APPENDIX.

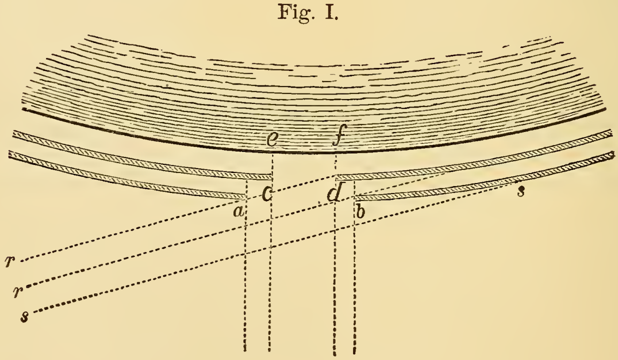
A.—Art. 107.

The exact period of the sun's rotation is not easily determined, because of the independent motions of the spots themselves. That they do have such motions is apparent from the fact that they differ from each other somewhat in their eastward velocity, and also that some of them move a little northward or southward while crossing the disk. Among the various results obtained by different observers, the lowest is about 25 days, and the highest about 25 days and 12 hours.

B.—Art. 111.

The perspective effect described in Art. 111, may perhaps be better understood by the aid of Fig. I., which represents a

Fig. I.



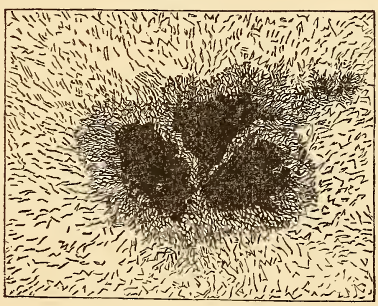
section of the sun through a spot. Let *ab* be the breadth of the opening in the outer stratum, *cd* that of the narrower one in the inner stratum. When this spot is seen near the middle

of the disk, we look into it almost at right angles to the surface, along the lines marked *p*, *p*, and can see *ef* of the denser part of the sun (which is the macula), and also some of the inner stratum on all sides of *cd;* and this is the umbra. But when the spot is very near the edge, we look along the lines *rr* through *ab*, and can see only that part of the inner stratum which is beyond *cd;* in other words, only that part of the umbra is seen which lies nearest to *s*, the edge of the disk.

C.—Art. 112.

The bright points and streaks which are generally visible over most of the sun's disk, giving it a mottled appearance, are called *faculæ* (*little torches*), and the dark specks among them are often called *pores.* The faculæ are described by some observers, as having the appearance of willow leaves crossing each other in all directions, and by others, as resembling rice

Fig. II.



grains, or bits of straw. They are most conspicuous at the edges of spots, and at places where spots are forming or closing up. Irregular bands of faculæ are frequently seen projecting themselves with great velocity over the area occupied by a spot, and even forming bridges entirely across it. Fig. II. imperfectly represents these appearances.

D.—Art. 115, and 228.

The combination of the spectroscope with the telescope has enabled astronomers to gain considerable additional knowledge

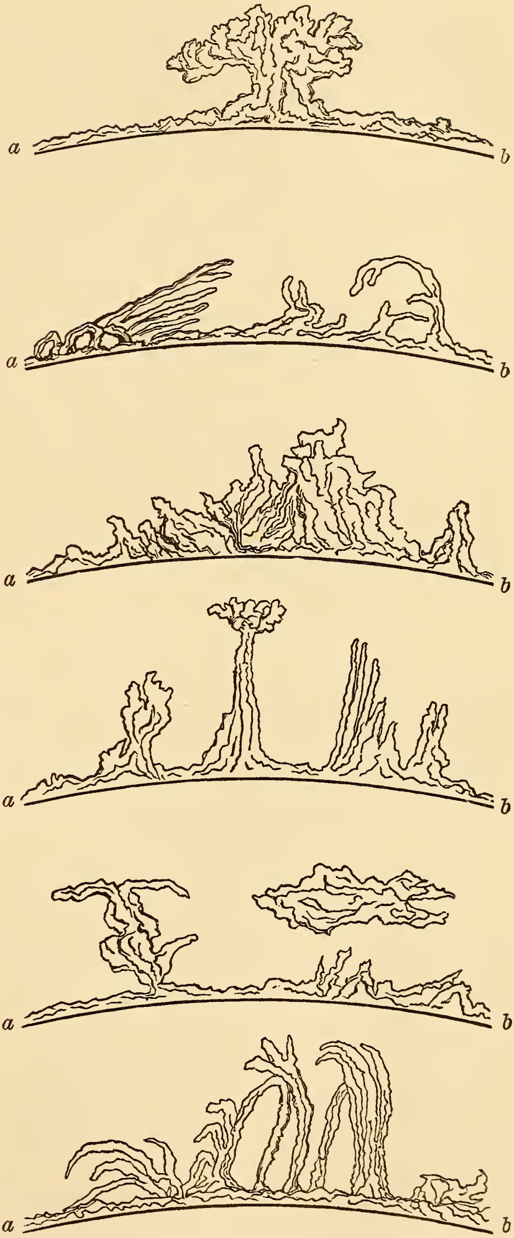
respecting the nature and condition of the sun's exterior. See Nat. Phil., Art. 398–400.

The dark lines of the solar spectrum show that the photosphere consists of the following substances in the gaseous state: sodium, calcium, magnesium, chromium, iron, copper, zinc, barium, nickel, hydrogen, etc. The intense light of the liquid parts below, shining through these gases, causes their spectrum lines, which would otherwise be bright colored lines, to become dark ones.

The same instrument has more recently proved the existence of a less luminous envelope outside of the photosphere. The appearance of irregular projecting masses of faint red light from behind the moon during a total solar eclipse, had previously led to the suspicion of such an atmosphere. See Art. 228, 3. By the use of modern instruments, not only can these *protuberances*, or *prominences* of reddish light be viewed at any time, but also the envelope itself can be traced entirely around the disk of the photosphere. This outer covering is called the *chromosphere*, because its spectrum exhibits colored instead of dark lines. This covering of red-hot gas consists largely of hydrogen, having an average depth of several hundred miles; but, being generally in a state of extreme commotion, its more elevated parts are from 50,000 to 100,000 miles high. The parts thus thrown upward by the terrific forces in operation there, are sometimes completely detached from the rest. The prominences of the chromosphere often resemble mountains, trees, flames, or clouds; but more frequently they assume fantastic forms wholly indescribable. These forms change very rapidly, indicating a motion of several thousands of miles in a single hour. In some cases there is evidence of rotary motion parallel to the surface of the sun—that is, there are vast whirlwinds of fire. In others, jets of red-hot hydrogen are spouted upward to the height of 50 or 60,000 miles. Fig. III. will convey some idea of the variety and singularity of the forms of the prominences of the chromosphere. The photosphere, or bright surface of the sun, is represented in each part of the figure by the curve *ab*.

The *corona*, which is white, and surrounds the chromosphere, extends considerably beyond its highest prominences. It is

Fig. III.



distinctly seen only during the totality of a solar eclipse, and its boundary is not at a uniform height on all sides, but varies irregularly from 100,000 to 200,000 miles in height from the photosphere.

A still fainter white light is seen during the time of a total solar eclipse, extending outward beyond the corona; this is called the *halo*. It was for a time suspected to be an effect produced by our own atmosphere. But there is increasing evidence furnished by recent eclipses, that it truly surrounds the sun. Its extent is very unequal on different sides, having in some places deep gaps reaching down to the corona, and in other parts extending upward to nearly twice the diameter of the sun.

E.—Art. 116.

There is an interesting connection between the periodicity of the solar spots, and that of magnetic disturbances on the earth. By a careful comparison of these phenomena, as observed and recorded through a period of nearly a hundred years, it is found that with the periodic increase and decrease of the amount of spot-surface on the sun's disk, there is a corresponding increase and decrease of terrestrial magnetic storms, indicated by the agitations of the needle and the occurrence of the aurora borealis. In each case, the maximum occurs at about the same time once in ten or eleven years, and the minimum at nearly corresponding times between. In general, there are frequent auroras and frequent disturbances of the needle in those years in which the sun exhibits the greatest spot-area; and when the solar spots are few and small, auroras are infrequent, and the needle is but little disturbed. Whatever may be the cause of spots on the sun, it is in the highest degree probable that the same cause produces these alternations in terrestrial magnetism.

F.—Art. 256.

There appears to be increasing evidence that there is at least one planet revolving within the orbit of Mercury. A round, dark spot has been repeatedly seen crossing the sun's disk; and a comparison of the dates of such observations leads to the belief that an inferior planet exists, whose periodic time is about 39 days. The name Vulcan, has been already given to the supposed planet. The existence of such a body, or else of a group of smaller bodies, has been for some time suspected, because of an unexplained motion of the perihelion of Mercury's orbit.

G.—Art. 357.

It does not necessarily follow from the reasoning in Art. 357, that *all* the light received from a comet is reflected. What is there stated would be true if a part is reflected, and another part originates in the comet itself. And the spectra of some

comets examined within a few years, furnish quite satisfactory evidence, that while they reflect the sun's light, they also radiate the light of some incandescent substances, either gaseous, or in the state of a comminuted solid.

H.—Art. 367.

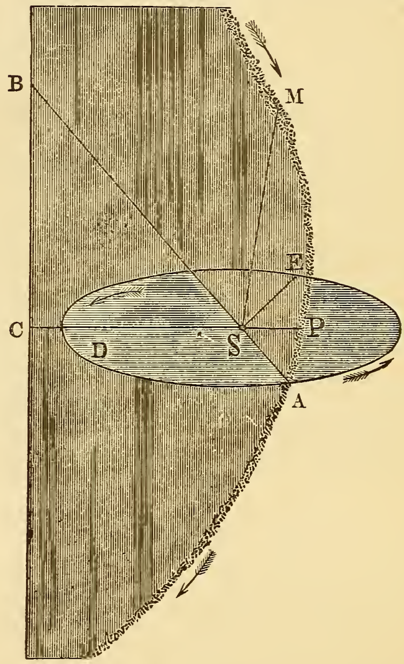
Biela's comet, which separated into two parts in 1846 while in sight from the earth, and which reappeared as two comets in 1852, has since then, it is believed, been partially or wholly divided into innumerable cometary fragments. For it has failed to appear at the times of its expected return since 1852, and in its stead there has been an unusual number of shooting stars coming into the earth's atmosphere at times and in directions corresponding to such a supposition. The path of the earth and that of Biela's comet so nearly intersected each other, that if the latter body has suffered the catastrophe supposed, it was to be expected that some of its fragments would meet the earth, and appear in its atmosphere as shooting stars.

I.—Art. 380.

The dissolution of a comet into a group or ring of meteors has taken place in other instances besides that mentioned in Appendix H. The annual meteoric shower of August 10th, comes from a ring which coincides with the orbit of Comet III., 1862. A small arc of this orbit is represented in Fig. IV., intersecting the earth's orbit at A, through which point the earth passes on the 10th of August. The planes of the two orbits intersect in the line AB, and their inclination, the angle ESM, is 64° 3′. The perihelion of the meteoric orbit is P; and PC drawn through the sun S, and produced, is the axis, which meets the aphelion at the distance of about 10,000,000,000 miles from the sun, or more than three times the distance of Neptune. The cometary fragments seem to be distributed around the whole circuit of the orbit, though unequally; so that the earth, when it passes across their path, always meets a few, and sometimes large numbers of them. The small comet, III., 1862, was probably the mere remainder of a large

comet, which has for ages been scattering its particles along its path; for the August shower has been known for a long time.

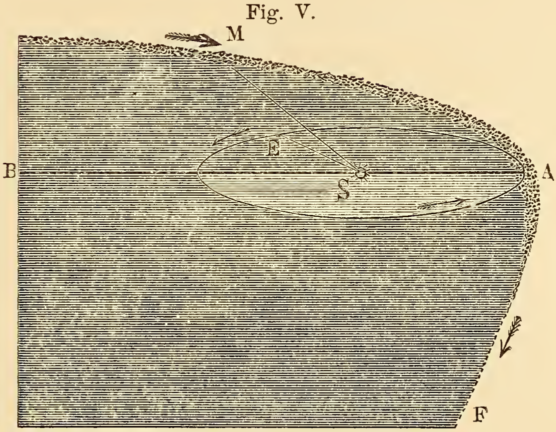
Fig. IV.



Another example of identity between a comet and a meteoric shower, is that of comet I., 1866, or Tempel's comet, and the shower of November 13th. The orbits have the same elements, and their periodic time is $33\frac{1}{4}$ years. In this case, the fragments of the comet, instead of occupying the whole circumference, are gathered into a group, of such length, however, that the earth strikes into it on three successive returns to the same place in its own orbit. Among the most brilliant displays of meteors from this group are those of 1799, 1833, and 1867. Fig. V. shows a short arc of this orbit, along which the meteors are moving in the direction of the arrows, in a group of varying thickness. It intersects the earth's orbit at A, the planes of the two being inclined at an angle of

17° 44′, represented by ESM. The aphelion is about as far from the sun as the orbit of Uranus. The meteors in both of

Fig. V.



the foregoing orbits have a retrograde motion, as the arrows show. Hence, the earth *meets* them, moving partly in a direction opposite to its own motion.

J.—Art. 392.

Spectroscopic observations made upon the brighter stars reveal the interesting fact, that, like the sun, they have a gaseous photosphere containing substances of the same nature as some of those existing on the earth. For example, *a* Tauri has hydrogen, sodium, magnesium, iron, mercury, and several other known elements in its gaseous exterior. Likewise, *a* Orionis, by the dark lines of its spectrum, is proved to have a constitution much like that of *a* Tauri. Hundreds of stars of the larger magnitudes have in like manner furnished some indications of the elements which compose them. But even the brightest stars shed so little light at our immense distance from them, that only the most conspicuous lines due to a given substance are visible. Yet the very exact coincidence of the few lines which can be seen, with those of the corresponding terrestrial elements, is considered as conclusive proof that these elements enter into the composition of such stars. It is

believed, therefore, that at least the brightest stars have a physical constitution similar to that of the sun of our own system. Their light, emanating from the denser central parts, passes through a luminous gaseous envelope, and by the dark lines thus exhibited, reveals the nature of the envelope.

There is no reason to suppose that the fainter stars, as a class, differ in their constitution from the brighter ones. They are too far off, however, to afford us sufficient light for ascertaining their true character by any means which have as yet been devised.

K.—Art. 393.

One of the most remarkable discoveries made by the use of the spectroscope is that of the motion of certain stars either *toward*, or *from* the solar system. A certain wave-length of light belongs to each point through the length of the spectrum. The waves of the red extremity are longest, and those of the violet extremity are shortest; and there is a regular gradation from one to the other. Nat. Phil., Art. 436. Hence, every line of the spectrum, since it has a fixed place, indicates precisely a certain wave-length corresponding to its location. Now, suppose that in the spectrum of a star, some of the stronger lines of a substance,—hydrogen, for example,—are discovered and known by their prominence and general locations, to be hydrogen lines; and suppose again, that when carefully examined under a high power, and compared with hydrogen artificially heated, that they are slightly displaced toward the violet end of the spectrum. This shows that the waves are a little shorter than those of hydrogen at rest. Such a displacement proves, therefore, that either the star is coming toward us, or we are approaching it; and the degree of displacement indicates the velocity of approach. A star, on the other hand, which shows a displacement of a set of lines from their true places toward the red extremity of the spectrum, is thereby known to be increasing its distance from the solar system. Thus, the star Sirius is discovered to be moving *from* the sun at the rate of nearly 30 miles per second.

The star may indeed be moving in a direction *oblique* to the

line joining it and the sun; but the spectroscopic displacement indicates only that component of the motion which is in the visual direction. The other component, if it exists, is what has been long recognized as the *proper* motion of the star; that is, its angular change of place, which cannot, however, be reckoned as a linear quantity, till the distance of the star from the solar system is ascertained.

L.—Art. 410.

So long as successive improvements in telescopes led occasionally to the resolving of a nebula into a cluster of stars, it remained uncertain whether all nebulæ consist of separate stars or not, until a new mode of investigation was discovered. Notwithstanding the great difficulties in the way of examining such faint objects with the spectroscope, the general question seems to be satisfactorily answered. All nebulæ, which have been hitherto resolved, exhibit a spectrum, apparently continuous, though dark lines too delicate to be discerned may exist. The bodies composing such nebulæ, therefore, consist of solid or liquid matter, which may or may not be surrounded by a gaseous envelope. Also, several of the nebulæ not yet resolved, show the same kind of spectrum; which indicates that these, too, are solid or liquid, but so remote as not to yield to the power of any telescope yet applied to them.

On the other hand, the larger part of irresolvable nebulæ, bright enough to be examined, form a spectrum which consists only of one, two, or three bright lines; and these generally coincide with those of some known gas. Thus, the ring nebula of Lyra gives a spectrum of one line, and that the brightest nitrogen line; and the great nebula of Orion, a spectrum of three lines,—one nitrogen, one hydrogen, and the third unknown.

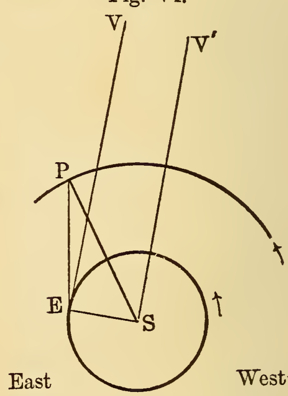
M.—Art. 264.

To identify any Planet.—When the observer has not the use of an Ephemeris he can find the approximate place of any of the primary planets by the following process:

Discarding the elliptical orbit, assume that the planet moves uniformly in a circular orbit. Multiply the mean daily motion both of the planet and the earth, as given in column VI, Table II, page 228, by the number of days that have elapsed since the beginning of the century, being careful to include the leap-days; to the product add the corresponding numbers in column VII. Divide by 360, so as to reject all the completed revolutions, and the remainders will be the mean heliocentric longitudes of the planet and of the earth.

In Fig. VI, let S be the sun; E, the earth; V, the vernal equinox as seen from the earth; V′, the same, as seen from the sun; and P the planet, the ecliptic being in the plane of the diagram. V′SP will be the heliocentric longitude of the planet; V′SE the heliocentric longitude of the earth; PSE the difference between them, and VEP the geocentric longitude of the planet.

Fig. VI.



In the triangle SEP, knowing SP and SE, either in astronomical units or in miles, and the angle ESP, we can compute the angle SEP by plane trigonometry. As SV′ and EV are parallel, SEV is the supplement of V′SE. By subtracting SEV from SEP, we have VEP, the required geocentric longitude of the planet.

If the diagram be held in the plane of the ecliptic, and the line EV pointed toward the vernal equinox, EP will point nearly in the direction of the planet. The inclination and eccentricity of the orbits of Mercury and the moon are so great that this method cannot be applied satisfactorily to finding their places.

SPECTROSCOPE.

(Fauth & Co., Manufacturers, Washington, D. C.)

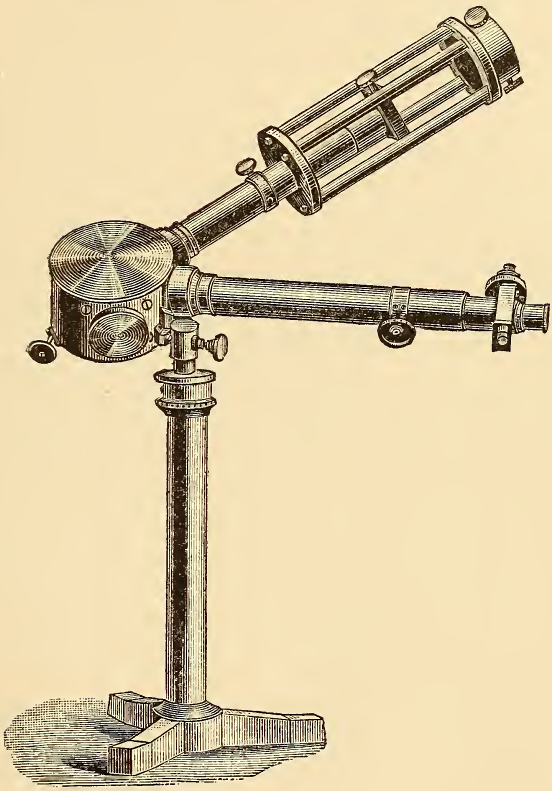


TABLE I.—THE CALENDAR.

A TABLE TO FIND THE DAY OF THE WEEK OF ANY GIVEN DATE, FROM THE YEAR 5000 B. TO THE YEAR 2700 OF THE CHRISTIAN ERA.

Centuries before Christ.												Centuries after Christ					
4800 4100 3400 2700 2000 1300 600	4700 4000 3300 2600 1900 1200 500	4600 3900 3200 2500 1800 1100 400	4500 3800 3100 2400 1700 1000 300	4400 3700 3000 2300 1600 900 200	5000 4300 3600 2900 2200 1500 800 100	4900 4200 3500 2800 2100 1400 700 0	New Style					1700 2100		1800 2200		1500 1900 2300	1600· 2000· 2400·	
							Old Style					0· 700· 1400· 2100·	100· 800· 1500· 2200·	200· 900· 1600· 2300·	300· 1000· 1700· 2400·	400· 1100· 1800· 2500·	500· 1200· 1900· 2600·	600· 1300· 2000· 2700·
C	D	E	F	G	A	B	0	28·	56·	84·	C	D	E	F	G	A	B
D	E	F	G	A	B	C	·1	·29	·57	·85	B	C	D	E	F	G	A
F	G	A	B	C	D	E	2	30	58	86	A	B	C	D	E	F	G
G	A	B	C	D	E	F	3	31	59	87	G	A	B	C	D	E	F
A	B	C	D	E	F	G	4·	32·	60·	88·	E	F	G	A	B	C	D
B	C	D	E	F	G	A	·5	·33	·61	·89	D	E	F	G	A	B	C
D	E	F	G	A	B	C	6	34	62	90	C	D	E	F	G	A	B
E	F	G	A	B	C	D	7	35	63	91	B	C	D	E	F	G	A
F	G	A	B	C	D	E	8·	36·	64·	92·	G	A	B	C	D	E	F
G	A	B	C	D	E	F	·9	·37	·65	·93	F	G	A	B	C	D	E
B	C	D	E	F	G	A	10	38	66	94	E	F	G	A	B	C	D
C	D	E	F	G	A	B	11	39	67	95	D	E	F	G	A	B	C
D	E	F	G	A	B	C	12·	40·	68·	96·	B	C	D	E	F	G	A
E	F	G	A	B	C	D	·13	·41	·69	·97	A	B	C	D	E	F	G
G	A	B	C	D	E	F	14	42	70	98	G	A	B	C	D	E	F
A	B	C	D	E	F	G	15	43	71	99	F	G	A	B	C	D	E
B	C	D	E	F	G	A	16·	44·	72·		D	E	F	G	A	B	C
C	D	E	F	G	A	B	·17	·45	·73		C	D	E	F	G	A	B
E	F	G	A	B	C	D	18	46	74		B	C	D	E	F	G	A
F	G	A	B	C	D	E	19	47	75		A	B	C	D	E	F	G
G	A	B	C	D	E	F	20·	48·	76·		F	G	A	B	C	D	E
A	B	C	D	E	F	G	·21	·49	·77		E	F	G	A	B	C	D
C	D	E	F	G	A	B	22	50	78		D	E	F	G	A	B	C
D	E	F	G	A	B	C	23	51	79		C	D	E	F	G	A	B
E	F	G	A	B	C	D	24·	52·	80·		A	B	C	D	E	F	G
F	G	A	B	C	D	E	·25	·53	·81		G	A	B	C	D	E	F
A	B	C	D	E	F	G	26	54	82		F	G	A	B	C	D	E
B	C	D	E	F	G	A	27	55	83		E	F	G	A	B	C	D

January. October.	February. March. November.	January, L. Y. April. July.	May.	June.	Feb., L. Y. August.	September. December.	A	B	C	D	E	F	G
1 8 15 22 29	5 12 19 26	2 9 16 23 30	7 14 21 28	4 11 18 25	6 13 20 27	3 10 17 24 31	Su	Sa	Fri	Th	W	Tu	Mo
2 9 16 23 30	6 13 20 27	3 10 17 24 31	1 8 15 22 29	5 12 19 26	7 14 21 28	4 11 18 25	Mo	Su	Sa	Fri	Th	W	Tu
3 10 17 24 31	7 14 21 28	4 11 18 25	2 9 16 23 30	6 13 20 27	1 8 15 22 29	5 12 19 26	Tu	Mo	Su	Sa	Fri	Th	W
4 11 18 25	1 8 15 22 29	5 12 19 26	3 10 17 24 31	7 14 21 28	2 9 16 23 30	6 13 20 27	W	Tu	Mo	Su	Sa	Fri	Th
5 12 19 26	2 9 16 23 30	6 13 20 27	4 11 18 25	1 8 15 22 29	3 10 17 24 31	7 14 21 28	Th	W	Tu	Mo	Su	Sa	Fri
6 13 20 27	3 10 17 24 31	7 14 21 28	5 12 19 26	2 9 16 23 30	4 11 18 25	1 8 15 22 29	Fri	Th	W	Tu	Mo	Su	Sa
7 14 21 28	4 11 18 25	1 8 15 22 29	6 13 20 27	3 10 17 24	5 12 19 26	2 9 16 23 30	Sa	Fri	Th	W	Tu	Mo	Su

EXPLANATION OF THE USE OF THE TABLE ON PAGES 226 AND 227.

Any year being given, either before or after Christ, Old or New Style, find the century at the top of the table on page 224, and the odd years in the middle column. The square of intersection contains the dominical letter for the year. Then look for the given day of the month in the table above, and the day of the week will be shown in the column under the dominical letter at the right hand, and in the same line with the day of the month. If the given year be a leap-year, and the month January or February, the day must be looked for under the head January, L. Y., or February, L. Y. A leap-year after Christ is marked by a dot on the right hand; one before Christ is marked on the left.

The *Dominical* (or *Lord's* day) letter for a given year means that which falls on Sunday, when th first seven letters of the alphabet are applied in order to the first seven days of January. Thus, if a year begins on Friday, the dominical letter of that year is C. The same letter marks every Sunday through the year, except in leap year, when there are two dominicals, one *before*, and the other *after*, the 29th of February.

Table II. Elements of the Planets.

I. NAME.	II. Symbol.	III. Relative Distances.	IV. Sidereal Revolution in days.	V. Synodical Revolut'n in days.	VI. Mean daily motion.	VII. Heliocentric Long. Jan.1,1801.	VIII. Heliocentric Long. Jan.1,1885.
					° ′ ″	° ′	° ′
Mercury..	☿	0.387098	87.969	115.877	4 5 32.6	213 20	82 24
Venus ...	♀	0.723332	224.701	583.921	1 36 7.8	11 33	206 54
Earth ...	♁	1.000000	365.256		59 8.3	100 39	100 18
Mars.....	♂	1.523691	686.980	779.936	31 26.7	64 23	296 45
Jupiter...	♃	5.202800	4332.554	398.884	4 59.3	112 15	147 00
Saturn...	♄	9.538800	10,759.106	378.092	2 0.6	135 20	81 28
Uranus...	♅	19.183380	30,686.246	369.656	42.4	177 48	179 52
Neptune..	♆	30.054370	60,228.072	367.485	21.7	227 18	52 13
Moon*....	◐	0.000259	27.321	29.530	13 10 36.6	118 17	105 13

Symbol.	IX. Inclinat'n of Orbit. Jan.1,1801.	X. Variation in 100 yrs.	XI. Eccentricity of Orbit, Jan.1,1801.	XII. Variation in 100 yrs.	XIII. Long. of A. Node Jan. 1, 1801.	XIV. Motion west. in 100 yrs.	XV. Long. of Perihel. Jan. 1, 1801.	XVI. Motion east. in 100 yrs.	XVII. Rotat'n in hours.
	° ′ ″	″	R.V.		° ′	′ ″	° ′	′ ″	
☿	7 0 9	+18	.205 515	+.000 004	45 57	13 4	74 21	9 43	24.09?
♀	3 23 28	—5	.006 811	—.000 063	74 51	32 24	128 43	5 12	23.35?
♁	0 0 0	0	.016 792	—.000 042			99 31	19 10	23.93
♂	1 51 6	—0.2	.093 307	+.000 090	48 0	41 26	332 23	26 13	24.62
♃	1 18 51	—23	.048 162	+.000 159	98 26	26 19	11 8	11 5	9.92
♄	2 29 36	—15	.056 151	—.000 312	111 56	32 23	89 9	32 11	10.24
♅	0 46 28	+3	.046 611	—.000 025	72 59	59 55	169	3 56	9.50?
♆	1 46 59		.008 719		130 6		46		
◐*	5 8 40		.054 908		13 53	1934°	266 10	4069°	708.73
Sun									608.

Symbol.	XVIII. Mean Diameter in miles.	XIX. Mean angular Diam.	XX. Relative Volume.	XXI. Relative Mass.	XXII. Rel'tive D'nsity.	XXIII. Rel'tive Gr'vity.	XXIV. Solar Light & Heat.	XXV. Velocity in Orbit in miles per sec.
		″						
☿	2,992	7	0.054	0.065	1.21	0.46	6.67	29.55
♀	7,660	17	0.880	0.769	0.85	0.82	1.91	21.61
♁	7,918		1.000	1.000	1.00	1.00	1.00	18.38
♂	4,211	9	0.248	0.111	0.73	0.39	.43	14.99
♃	86,000	37	1350.	311.953	0.24	2.64	.037	8.06
♄	70,500	16	689.	93.329	0.13	1.18	.011	5.95
♅	31,700	4	75.	14.460	0.22	0.90	.003	4.20
♆	34,500	3	102.	16.862	0.20	0.89	.001	3.36
◐	2,161	1866	0.020	0.012	0.60	0.16	1.000	0.63
Sun	860,000	1924	1295000.	326800.	0.25	27.71		

* Mean geocentric values.

Table III. Elements of the Satellites.

THE MOON.

Mean distance from the earth, (miles)	238.820
Mean sidereal revolution, (days)	27.32166
Mean synodical revolution, (days)	29.53058
Mean revolution of nodes, (days)	6793.39108
Mean revolution of apsides (days)	3232.57534
Mean inclination of orbit to ecliptic	5° 8′ 44″
Eccentricity of orbit	0.054908
Mean diameter of moon, (miles)	2161
Diameter, (earth's = 1)	0.2730
Surface, (earth's = 1)	$\frac{1}{13}$ or 0.0745
Volume, (earth's = 1)	$\frac{1}{49}$ or 0.0203
Density, (earth's = 1)	$\frac{2}{3}$ or 0.6052
Mass, (earth's = 1)	$\frac{1}{81.4}$ or 0.0123
Gravity, (earth's = 1)	$\frac{1}{6}$ or 0.165

	Number.	Sidereal Revolutions.				Distance in equatorial radii of Planet.	Distance in miles.	Diameter in miles.
		d.	h.	m.	s.			
Satellites of Jupiter.	1	1	18	27	34	6.04853	260000	2365
	2	3	13	14	36	9.62347	414000	2123
	3	7	3	42	33	15.35024	661000	3471
	4	16	16	31	50	26.99835	1162000	2966
Satellites of Saturn.	1	0	22	37	23	3.3607	122000	
	2	1	8	53	7	4.3125	157000	
	3	1	21	18	26	5.3396	194000	
	4	2	17	41	9	6.8398	248000	
	5	4	12	25	11	9.5528	347000	1163
	6	15	22	41	25	22.1450	804000	2908
	7	21	7	7	41	26.7834	973000	
	8	79	7	53	40	64.3590	2338000	1745
Satellites of Uranus.	1	2	12	29	21	7.40	123000	
	2	4	3	28	8	10.31	172000	
	3	8	16	56	31	16.92	282000	
	4	13	11	7	13	22.56	376000	
Satellite of Neptune.		5	21	2	43	12.	222000	

TABLE IV. MEAN PLACES OF PRINCIPAL STARS ; 1885, JAN. 0.

No.	STAR'S NAME.	Mag.	Right Ascension.	Annual Var.	North Polar Distance.	Annual Var.
			H. M. S.	S.	° ′ ″	″
1	α Andromedæ (Alpherat).	2.0	0 2 26	+3.09	61 32 40	−19.89
2	γ Pegasi (Algenib)......	2.7	0 7 18	+3.08	75 27 21	−20.02
3	Nebula in Andromeda.	....	0 35	... ...	49 23	....
4	β Ceti..................	2.	0 37 49	+3.01	108 37 5	−19.80
5	β Andromedæ..........	2.3	1 3 17	+3.34	64 59 22	−19.17
6	α Ursæ Minoris (Polaris).	2.	1 16 37	+22.46	1 18 16	−18.94
7	α Eridani (Achernar)....	1.	1 33 25	+2.23	147 49 16	−18.36
8	α Arietis................	2.	2 0 41	+3.37	67 4 55	−17.18
9	Cluster in Perseus....	....	2 10	....	33	....
10	ο Ceti (Mira)...........	Var.	2 13	....	93 32	....
11	α Ceti.....	2.3	2 56 16	+3.12	86 21 43	−14.32
12	β Persei (Algol).........	Var.	3 0 41	+3.88	49 29 18	−14.13
13	α Persei.......	2.	3 16 7	+4.25	40 32 57	−13.11
14	η Tauri (Pleiades).......	3.	3 40 38	+3.55	66 15 6	−11.40
15	γ Tauri (Hyades).......	4.	4 13 15	+3.40	74 39 3	−8.98
16	α Tauri (Aldebaran).....	1.	4 29 19	+3.43	73 43 22	−7.53
17	α Aurigæ (Capella)......	1.	5 8 11	+4.42	44 7 14	−4.06
18	β Orionis (Rigel)........	1.	5 9 0	+2.88	98 20 7	−4.42
19	δ Orionis...............	Var.	5 26 8	+3.06	90 23 7	−2.94
20	Nebula in Orion......	....	5 29	.... ..	95 29	....
21	α Orionis.........	Var.	5 48 56	+3.24	82 36 55	−0.97
22	α Argus (Canopus)......	1.	6 21 24	+1.33	142 37 59	+1.86
23	α Canis Maj. (Sirius)....	1.	6 40 4	+2.64	106 33 33	+4.69
24	α Geminorum (Castor)...	1.7	7 27 15	+3.84	57 51 37	+7.53
25	α Canis Min. (Procyon)..	1.	7 33 16	+3.14	84 28 52	+8.97
26	β Geminorum (Pollux)..	1.3	7 38 16	+3.67	61 41 50	+8.40
27	Cluster, Præsepe......	...	8 20	....	69 50	....
28	α Hydræ...............	2.	9 21 56	+2.94	98 9 38	+15.44
29	α Leonis (Regulus)......	1.3	10 2 14	+3.20	77 28 16	+17.46
30	η Argus (variable).......	1–6	10 40 36	+2.31	149 4 48	+18.86
31	α Ursæ Majoris (Dubhe).	2.	10 56 37	+3.75	27 37 42	+19.35
32	α Crucis................	1.	12 20 11	+3.27	152 27 42	+20.01
33	α Virginis (Spica).......	1.	13 19 8	+3.15	100 33 38	+18.91
34	α Bootis (Arcturus)......	1.	14 10 25	+2.73	70 13 7	+18.89
35	β Ursæ Minoris...	2.	14 51 3	−0.23	15 22 28	+14.72
36	α Coronæ Borealis.......	2.	15 29 49	+2.54	62 53 52	+12.32
37	α Scorpii (Antares)......	1.3	16 22 21	+3.67	116 10 32	+8.32
38	α Ophiuchi.............	2.	17 29 36	+2.78	77 21 20	+2.89
39	α Lyræ (Vega)..........	1.	18 33 3	+2.03	51 19 22	−3.15
40	Annular Nebula, Lyra.	. .	18 49	....	57 7	....
41	α Aquilæ (Altair).......	1.3	19 45 10	+2.92	81 26 5	−9.25
42	Dumb-bell Nebula....	....	19 54	....	67 36	....
43	α Delphini.............	3.7	20 34 18	+2.79	74 29 36	−12.50
44	α Cygni................	1.7	20 37 31	+2.04	45 7 49	−12.71
45	12 Year Catalogue, 1879..	6.	20 52 46	−2.53	9 52 47	−13.70
46	61 Cygni................	5.	21 1 45	+2.68	51 48 56	−17.52
47	α Piscis Aus. (Fomalhaut)	1.3	22 51 17	+3.32	120 13 53	−18.99
48	α Pegasi (Markab)......	2.	22 59 2	+2.98	75 24 48	−19.30

TABLE V. THE PLANETOIDS.

No.	NAME.	Mean daily motion	Log. of mean dist'nce.	No.	NAME.	Mean daily motion	Log. of mean dist'nce.
1	Ceres.........	770.8332	.442031	51	Nemausa.....	975.6485	.373809
2	Pallas........	769.7324	.442444	52	Europa.......	651.2204	.490852
3	Juno.........	812.9059	.426644	53	Calypso.......	837.8551	.417891
4	Vesta.........	976.7787	.373474	54	Alexandra....	794.1220	.433412
5	Astræa.......	857.9269	.411037	55	Pandora.......	774.3196	.440724
6	Hebe.........	939.3696	.384780	56	Melete........	847.7131	.414505
7	Iris..........	962.5806	.377713	57	Mnemosyne...	635.2707	.498032
8	Flora.........	1086.3309	.342696	58	Concordia....	799.5964	.431423
9	Metis	962.3390	.377786	59	Elpis.........	793.9788	.433465
10	Hygeia.......	637.1610	.497171	60	Echo.........	958.1112	.379060
11	Parthenope...	923.6604	.389663	61	Danaë........	687.6656	.475086
12	Victoria......	994.8347	.368139	62	Erato.........	642.5658	.494726
13	Egeria........	857.9451	.411031	63	Ausonia......	956.1364	.379658
14	Irene.........	852.4385	.412896	64	Angelina.....	806.8077	.428824
15	Eunomia......	825.4550	.422209	65	Cybele........	558.3014	.535425
16	Psyche.......	710.9629	.465440	66	Maia.........	824.7087	.422471
17	Thetis........	911.3975	.393532	67	Asia	941.5410	.384111
18	Melpomene...	1020.1198	.360903	68	Leto..........	765.2766	.444125
19	Fortuna......	929.6590	.387788	69	Hesperia......	689.8760	.474157
20	Massalia......	949.0444	.381813	70	Panopea......	839.0994	.417462
21	Lutetia.......	933.5544	.386578	71	Niobe	774.6491	.440601
22	Calliope......	715.6529	.463536	2	Feronia.......	1040.1026	.355287
23	Thalia	833.0737	.419548	73	Clytia........	815.4003	.425757
24	Themis.......	640.1662	.495809	74	Galatea.......	765.7921	.443930
25	Phocea.......	954.6367	.380112	75	Eurydice.....	813.0315	.426599
26	Proserpina ...	819.6847	.424240	76	Freia.........	560.9129	.534074
27	Euterpe	986.6944	.370549	77	Frigga.......	814.1350	.426206
28	Bellona.......	766.0691	.443825	78	Diana........	836.5607	.418339
29	Amphitrite....	869.0352	.402312	79	Eurynome....	928.8736	.388033
30	Urania.......	975.1642	.373952	80	Sappho.......	1020.0052	.360936
31	Euphrosyne..	635.1686	.498079	81	Terpsichore...	736.1744	.455350
32	Pomona......	852.5880	.412845	82	Alcmene......	772.7477	.441312
33	Polyhymnia..	732.0291	.456985	83	Beatrice......	936.6007	.385635
34	Circe.........	806.1634	.429055	84	Clio	977.8108	.373167
35	Leucothea....	685.1834	.476133	85	Io............	821.4080	.423632
36	Atalanta......	780.0110	.438604	86	Semele.......	649.2352	.491736
37	Fides...... ...	826.0660	.421994	87	Sylvia........	543.7017	.543097
38	Leda.........	782.5641	.437657	88	Thisbe........	770.2917	.442234
39	Lætitia.......	769.9967	.442345	89	Julia	870.8412	.406712
40	Harmonia.....	1039.3353	.355500	90	Antiope.......	636.1509	.497631
41	Daphne	770.1514	.442287	91	Ægina........	851.2296	.413306
42	Isis...	930.9057	.387401	92	Undina.......	622.3687	.503972
43	Ariadne......	1084.1384	.343281	93	Minerva......	775.6388	.440231
44	Nysa	941.3988	.384155	94	Aurora.......	630.8636	.500047
45	Eugenia......	789.0034	.435285	95	Arethusa.....	659.2278	.487314
46	Hestia........	883.9660	.402381	96	Ægle.........	666.2189	.484260
47	Aglaia	725.9827	.459386	97	Clotho........	813.1887	.426543
48	Doris.........	646.1069	.493134	98	Ianthe	805.3700	.429341
49	Pales.........	653.3922	.489888	99	Dice	758.6620	.446640
50	Virginia......	822.4986	.423247	100	Hecate........	652.0664	.490476

TABLE V. (continued). THE PLANETOIDS.

No.	NAME.	Mean daily motion	Log. of mean dist'nce.	No.	NAME.	Mean daily motion	Log. of mean dist'nce.
101	Helena.......	853.6127	.412497	151	Abundantia...	850.7264	.413478
102	Miriam.......	816.7370	.425283	152	Atala.........	639.0187	.496329
103	Hera..	799.0675	.431615	153	Hilda.........	451.5802	.596847
104	Clymene	634.4466	.498408	154	Bertha.......	622.3629	.503975
105	Artemis......	971.0795	.375168	155	Scylla.........	713.7875	.464292
106	Dione........	629.5650	.500644	156	Xantippe.....	670.2300	.482522
107	Camilla	545.4463	.542170	157	Dejanira......	854.8040	.412092
108	Hecuba.......	616.3698	.506777	158	Coronis.......	730.5502	.457571
109	Felicitas.. ...	802.0510	.430555	159	Æmilia.......	647.7291	.492411
110	Lydia.........	785.1449	.436704	160	Una..........	787.1915	.435951
111	Ate......... ...	849.9278	.413749	161	Athor........	970.0005	.375489
112	Iphigenia.....	934.4391	.386304	162	Laurentia	673.1350	.481270
113	Amalthea.....	968.1836	.376032	163	Erigone.......	981.1480	.372181
114	Cassandra	810.8275	.427385	164	Eva..........	829.6880	.420728
115	Thyra........	965.9609	.376698	165	Loreley	642.0938	.494938
116	Sirona........	771.4040	.441816	166	Rhodope	803.0021	.430193
117	Lomia........	686.0326	.475775	167	Urda.........	614.4750	.507668
118	Peitho........	931.6917	.387156	168	Sibylla	570.0346	.529402
119	Althea	855.5046	.411856	169	Zelia.........	978.5025	.372963
120	Lachesis......	644.3548	.493921	170	Myrrha.......	868.8279	.407382
121	Hermone.....	551.5624	.538941	171	Ophelia.......	635.5487	.497905
122	Gerda........	615.5690	.507153	172	Baucis	966.3982	.376567
123	Brunhild......	801.8499	.430609	173	Ino...........	780.2369	.438520
124	Alcestis.......	832.0020	.419921	174	Phædra	732.1255	.456947
125	Liberatrix....	780.7231	.438339	175	Andromache. .	541.0099	.544534
126	Velleda	930.9792	.387377	176	Idunna.......	622.6360	.503848
127	Johanna......	775.3364	.440344	177	Irma....	774.6923	.440585
128	Nemesis......	777.4964	.439538	178	Belisaria......	920.0970	.390782
129	Antigone.....	727.2294	.458890	179	Clytemnestra..	692.2257	.473172
130	Electra.......	642.9388	.494558	180	Garumna.....	787.4120	.435870
131	Vala	942.2999	.383878	181	Eucharis......	644.0102	.494075
132	Æthra........	846.3646	.414966	182	Elsbeth	944.0487	.383341
133	Cyrene.......	663.5850	.485406	183	Istria.........	756.3767	.447526
134	Sophvosyne...	864.5740	.408803	184	Deipeia.	623.2669	.503555
135	Hertha.......	638.1149	.385167	185	Eunike.......	783.0772	.437468
136	Austria.......	1026.3921	.359129	186	Celuta........	977.1085	.373376
137	Melibœa......	641.8566	.495046	187	Lamberta.....	782.3914	.437722
138	Tolosa........	926.0192	.388924	188	Menippe......	748.8250	.450417
139	Juewa........	765.7567	.443944	189	Phthia........	924.9882	.389247
140	Siwa.........	789.1234	.436343	190	Ismene.......	454.0674	.595257
141	Lumen.......	814.5161	.426071	191	Kolga........	722.4983	.460780
142	Polana.......	942.8756	.383701	192	Nausika......	952.5933	.380673
143	Adria.........	773.0080	.441216	193	Ambrosia.....	858.2960	.410913
144	Vibilia........	821.2984	.423670	194	Proene........	836.9383	.418209
145	Adeona.......	815.4470	.425740	195	Eurycleia.....	728.9100	.458222
146	Lucina	789.8850	.434962	196	Philomela....	653.8370	.489692
147	Protogeneia...	638.6654	.496488	197	Arete	780.9746	.438246
148	Gallia........	769.5145	.442526	198	Ampella......	922.9325	.389894
149	Medusa.......	1139.1950	.328939	199	Byblis........	618.1730	.505931
150	Nuwa........	689.3407	.474381	200	Dynamene....	783.2609	.437400

TABLE V. (continued). THE PLANETOIDS.

No.	NAME.	Mean daily motion	Log. of mean dist'nce.	No.	NAME.	Mean daily motion	Log. of mean dist'nce.
201	Penelope. . . .	809.9320	.427706	218	Bianca.	817.2760	.425090
202	Chryseis.	655.0080	.489173	219	Thusnelda. . . .	982.3480	.371828
203	Pompeia.	782.7813	.437577	220	(Mar. 19, 1881.)	974.5910	.374123
204	Callisto.	812.0185	.426960	221	(Jan. 18, 1882.)	678.2950	.479058
205	Martha	766.6919	.443590	222		645.2880	.493502
206	Hersilia.	. . . .	. . . .	223		650.1600	.491324
207	Hedda.	1027.3643	.358855	224		826.1800	.421954
208	Lacrimosa. . . .	729.1020	.458146	225		568.9810	.529939
209	Dido.	637.0860	.497206	226		792.4160	.434036
210	Isabella.	780.0227	.438599	227	Philosophia. . .	626.1270	.502229
211	Isolda.	667.2952	.483792	228		1084.5100	.343182
212	Medea.	644.9370	.493660	229		567.8920	.530494
213	Lilæa.	779.8090	.438679	330	Athamantis. . .	963.8230	.377340
214	Aschera.	840.9460	.416826	231	(Sept. 10, 1882.)	701.3150	.469396
215	Œnone	770.4950	.442158	232	Russia	870.2300	.406915
216	Cleopatra.	759.6820	.446250	233	(May 11, 1883.)	. . . .	. . . .
217	Eudora.	665.7647	.484457				

CHRONOGRAPH.

(Fauth & Co., Manufacturers, Washington, D. C.)

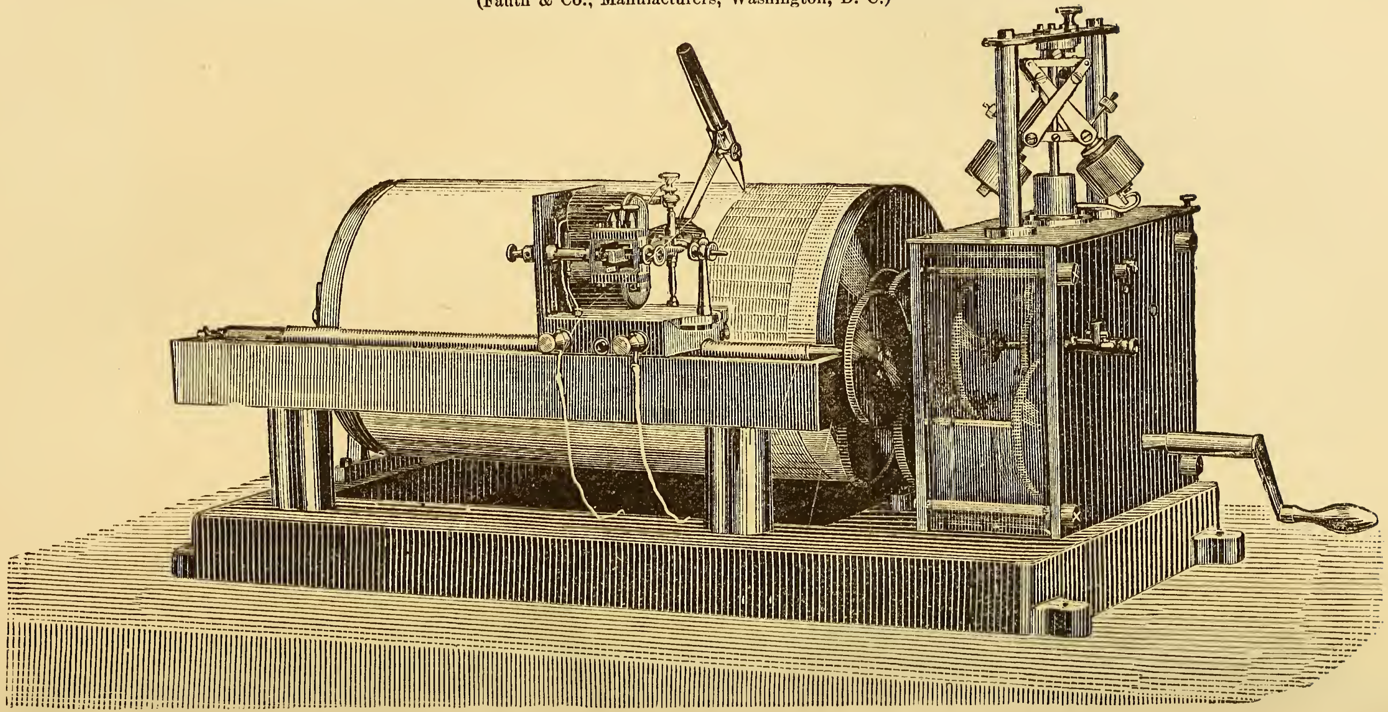


PLATE I.

COMET OF 1843.

PLATE II.

COMET OF 1858.—NEBULÆ.

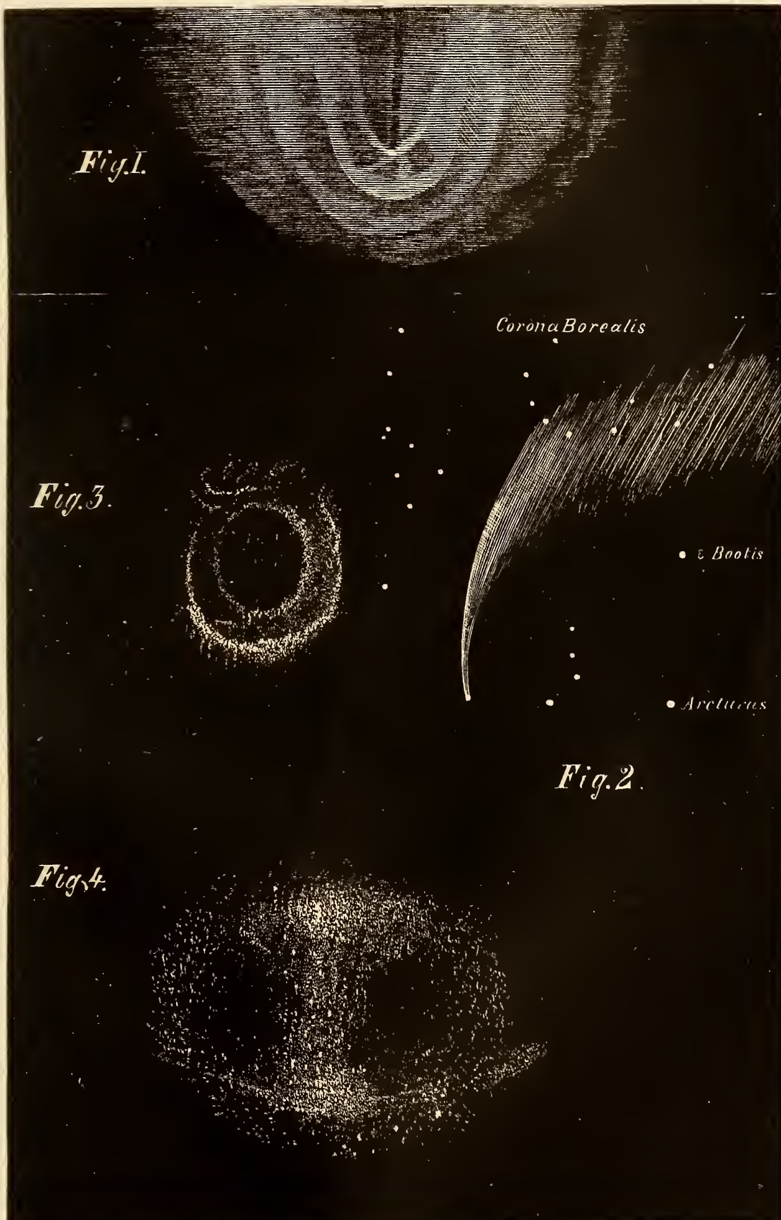
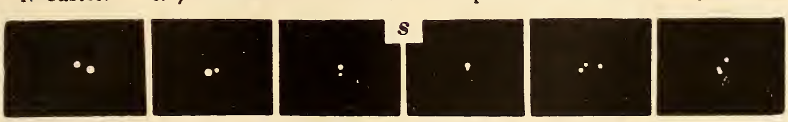


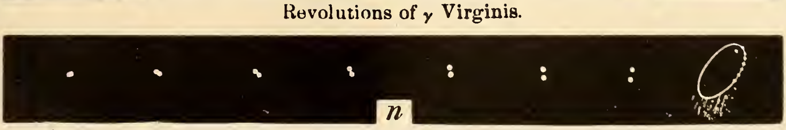
PLATE III.

PART OF GALAXY.—DOUBLE STARS

1. Castor. 2. γ Leonis. 3. 39 Drac. 4. λ Oph. 5. 11 Monoc. 6. ζ Cancri.



Revolutions of γ Virginis.



1837. 1838. 1839. 1840. 1845. 1850. 1860. Orbit.

PLATE IV

CLUSTERS.—NEBULÆ.